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Investigating convergence of subseries of harmonic series with respect to corresponding gap sequences

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It is well known that the harmonic series $\sum_{n=1}^{\infty}\frac{1}{n}$ is divergent. There have been numerous studies investigating the convergence of subseries $\sum_{n=1}^{\infty}\frac{1}{p_n}$ of harmonic series when p_n takes a specific form. Another possible way to study the convergence of such a subseries is in terms of the corresponding gap sequence $(d_n)_{n=1}^{\infty}$, where $d_n = p_{n+1} - p_n$.

For a given subseries $\sum_{n=1}^{\infty}\frac{1}{p_n}$ of harmonic series with gap sequence $(d_n)_{n=1}^{\infty}$ we consider any permutation ρ of the positive integers and define a new sequence $(e_n)_{n=1}^{\infty}$ by $e_n = d_{\rho(n)}$ for each n, making $(e_n)_{n=1}^{\infty}$ a rearrangement of $(d_n)_{n=1}^{\infty}$. For each n, we now have a new sequence of positive integers $(q_n)_{n=1}^{\infty}$ that corresponds to the gap sequence $(e_n)_{n=1}^{\infty}$.

In this study we will be trying to determine conditions required for $(d_n)_{n=1}^{\infty}$ so that the subseries $\sum_{n=1}^{\infty}\frac{1}{p_n}$ and $\sum_{n=1}^{\infty}\frac{1}{q_n}$ of harmonic series would have the same behavior of convergence for every permutation ρ. For example, it is shown that if $(d_n)_{n=1}^{\infty}$ is strictly increasing then $\sum_{n=1}^{\infty}\frac{1}{p_n}$ is convergent and for every permutation ρ the subseries $\sum_{n=1}^{\infty}\frac{1}{q_n}$ is also convergent. On the other hand if $(d_n)_{n=1}^{\infty}$ assumes a certain value *c* for infinitely many times and $\sum_{n=1}^{\infty}\frac{1}{p_n}$ is convergent, not every subseries $\sum_{n=1}^{\infty}\frac{1}{q_n}$ obtained by rearranging $(d_n)_{n=1}^{\infty}$ is convergent.

Keywords: Subseries of harmonic series, Convergence, Gap sequence

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