

The Simplest Proof of Fermat's Last Theorem

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It is well known that proof of Fermat's last theorem is very difficult and the main objective of this paper is to point out that if we make the Fermat equation a rational number equation then the proof becomes simple.

Fermat's last theorem for $n(> 2)$ can be stated thus: There are non-trivial integers x, y, z satisfying the equation $z^n = y^n + x^n$, $(x, y) = 1, n > 2$. Rearranging and relabeling the integers, we can assume without loss of generality that $z, y, x > 0$. From the above equation, we obtain $g^n = h^n + 1$ where $g = \frac{z}{x}, h = \frac{y}{x}, g, h > 0$. From this rational number equation, we obtain the following nature of d and the equations satisfied by d .

$(g - h)[g^{n-1} + g^{n-2}h + \dots + gh^{n-2} + h^{n-1}] = g^n - h^n = 1$, and if $g - h = d$, we have $g^{n-1} + g^{n-2}h + \dots + gh^{n-2} + h^{n-1} = \frac{1}{d} > 0$, $d > 0$. Now, we obtain the equation in d .

$d([(d + h)^{n-1} + h(d + h)^{n-2} + \dots + (d + h)h^{n-2} + h^{n-1}]) = 1$. From which we get the equation $d^n + nhd^{n-1} + \dots + \left(\frac{n(n-1)}{2}\right)h^{n-2}d^2 + nh^{n-1}d - 1 = 0$ since $h, d > 0$, we get

$\left(\frac{n(n-1)}{2}\right)h^{n-2}d^2 + nh^{n-1}d - 1 < 0$. Let us consider the property of this quadratic writing it as

$-\left(\frac{n(n-1)}{2}\right)h^{n-2}d^2 - nh^{n-1}d + 1 > 0$ which is a quadratic expression in d and the expression is positive and therefore the discriminant of it should be negative i.e. $n^2h^{2n-2} + 2n(n-1)h^{n-2} < 0$. Which never holds and since $h > 0$ and that the original Fermat equation is not satisfied by non trivial integer triples x, y, z .

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