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Simple and Short Proof of Fermat's Last Theorem for n=7

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Fermat's last theorem for $n = 7$ was first proved[1],[2] by Lame in 1839 and some others developed proofs later. None of the proofs is easy to understand and extended for all primes. Germain Sophie developed an important theorem that if p is a prime and 2p+1 is not a prime, then the equation

$$z^p = y^p + x^p, (x, y) = 1 \quad (1)$$

has two types of solutions $xyz \equiv 0 \ (mod p)$ and $xyz \not\equiv 0 (mod p)$. Hence, the equation corresponding to $p = 7$ also has the two solutions mentioned above if we assume it has solutions. Using this property, a simpler proof [3] has been developed. Our main interest is to give a proof for this case which can be used for any odd prime easily.

In proving Fermat's last theorem for n=7, we make use of a new approach. First, we transform the Fermat equation corresponding to FLT for $n = 7$ to a rational number equation and then we use very elementary mathematics to prove the theorem.

Fermat's last theorem for n=7 can be stated thus;

There are no non trivial integers x, y, z satisfying the equation

$$z^7 = y^7 + x^7, (x, y) = 1 \quad (2)$$

It is clear that we can assume, without loss of generality ,that $z > y > x > 0$. The equation (2) can be transformed to

$$g^7 = h^7 + 1 \quad (3)$$

where $g = \frac{z}{y}$ and $h = \frac{y}{x}$

$$(g - h)[g^6 + hg^5 + h^2g^4 + \cdots \dots \dots + h^5g + h^6] = 1,$$

$$(d)[(h + d)^6 + h(h + d)^5 + h^2(h + d)^4 + \cdots \dots \dots + h^5(h + d) + h^6] = 1 \quad (4)$$

$$(d)[h^6 + d^6 + 6hd^5 + \cdots \dots .. + 6h^5d + h^6 + hd^5 + 5h^2d^4 + \cdots + 5h^5d + h^2d^4 + h^6 + 4h^3d^3 + \cdots \dots . + 4h^5d + \cdots \dots \dots + h^6 + h^5d + h^6] = 1 \quad (5)$$

$$[d^7 + 7hd^5 + + \cdots \dots \dots + (6 + 5 + 4 + \cdots \dots . + 1)d^2h^5 + 7h^6d - 1] = 0 \quad (6)$$

If $g - h = d$, $[g^6 + hg^5 + h^2g^4 + \cdots \dots \dots + h^5g + h^6] = \frac{1}{d} > 0$ and therefore $d > 0$.

From (6) , we get $21h^5d^2 + 7h^6d - 1 < 0$, Therefore $-21h^5d^2 - 7h^6d + 1 > 0$ and this means that the quadratic in d is positive and hence the discriminant of it should be negative. In other words, $49h^{12} + 84h^5 < 0$, which is an apparent contradiction. Therefore, the equation does not hold and Fermat's last theorem for n=7 follows.

We transformed the Fermat equation corresponding to FLT for n=7 to a rational number equation first. Next, we used elementary mathematics to derive a strong contradiction and thereby we were able conclude that the original equation has no integer solution. It should be emphasized that our proof is very simple and short and can be extended to any prime number p.

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