

On sum formulas for Mordell - Tornheim zeta values

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The multiple zeta values are real numbers which are studied by many people in different fields. The multiple zeta values with depth 1 are the Riemann zeta values. The sum formulas are considered as one of the most famous relations among multiple zeta values. In our research, we study a slightly different type of sums known as Mordell-Tornheim zeta values. Mordell-Tornheim zeta values can be expressed as a rational linear combination of multiple zeta values with same depth and weight.

We have obtained new sum formulas for Mordell-Tornheim zeta values in the case of depth 2 and 3, expressing the sums as single multiples of Riemann zeta values. Moreover, we introduce reciprocity relations between the Mordell-Tornheim series of even arguments with depth 3 in terms of double and triple zeta values by using integrals of products of Bernoulli polynomials.

Keywords: Mordell-Tornheim zeta values, Multiple zeta values, Riemann zeta values, Sum formulas