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A class of s-step non-linear iteration scheme based on projection method for s-stage Runge-Kutta method

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A variety of linear iteration schemes with reduced linear algebra costs have been proposed to solve the non-linear equations arising in the implementation of implicit Runge-Kutta methods as an alternative to the modified Newton iteration scheme. In this paper, a class of s-step non-linear scheme based on projection method is proposed to accelerate the convergence rate of those linear iteration schemes. The s-step scheme is given by

$$Y^m = Y^1,$$

$$\mu_i^m = \frac{\left[(Q\otimes I_n)(I_{sn}-hA\otimes J)E_i^m\right]^H (Q\otimes I_n)D(Y^{(i)})}{\left[(Q\otimes I_n)(I_{sn}-hA\otimes J)E_i^m\right]^H \left[(Q\otimes I_n)(I_{sn}-hA\otimes J)E_i^m\right]},$$

$$Y^{(i+1)} = Y^{(i)} + \mu_i^m E_i^m, \quad i = 1,2,3,\ldots,$$

$$Y^{m+1} = Y^{(s+1)}, \quad m = 1,2,3,\ldots,$$

where μ^m is a scalar, $E_i^m = O\oplus O\oplus\cdots\oplus\varepsilon_i^m\oplus O\oplus\cdots\oplus O$, O the zero vector. In this scheme, sequence of numerical solutions is updated after each sub-step is completed. That is $Y^{(i)} = y_1^{m+1}\oplus y_2^{m+1}\oplus\cdots\oplus y_{i-1}^{m+1}\oplus y_i^m\oplus y_{i+1}^m\oplus\cdots\oplus y_s^m$ for $i = 1,2,3,\ldots,s$.
The efficiency of this scheme was examined when it is applied to the linear scalar problem $y' = qy, q\in\mathbb{C}$ with rapid convergence required for all $z = hq$ in the left half complex plane, where h is a step size, and obtained the iteration matrix of this scheme. The non-singular matrix Q should be chosen to minimize the maximum of the spectral radius of the iteration matrix over the left half complex plane. For 2-stage Gauss method, upper bound for the spectral radius of the iteration matrix was obtained in the left half complex plane. In this approach, it is difficult to handle the 3-stage Gauss method and 4-stage Gauss methods. We transform the coefficient matrix and the iteration matrix to a block diagonal matrix. The result for *s*=2 is applied to other methods when *s*>2. Finally, some numerical experiments are carried out to confirm the obtained theoretical results. Numerical result shows that, the proposed class of non- linear iteration scheme accelerates the convergence rate of the linear iteration scheme that we consider for the comparison in this work. It will be possible to apply the proposed class of non-linear scheme to accelerate the rate of convergence of other linear iteration schemes.

Keywords: Implementation, Non-linear scheme, Projection method, Rate of convergence, Stiff systems