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Bound on Weierstrass vertices of graphs

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Chip-firing games and divisor theory on finite, connected, undirected and unweighted graphs have been studied as analogs of divisor theory on Riemann Surfaces and algebraic curves. As part of this theory, a version of the one-dimensional Riemann-Roch theorem was introduced for graphs by Matt Baker in 2007. Properties of algebraic curves that have been studied can be applied to study graphs by means of the divisor theory of graphs. In this research, we investigate the property of a vertex of a graph as having the Weierstrass property in analogy to the theory of Weierstrass points on algebraic curves. The weight of the Weierstrass vertices is then calculated in a manner analogous to the algebraic curve case. Although there are many graphs for which all vertices are Weierstrass vertices, there are bounds on the total weight of the Weierstrass vertices as a function of the arithmetic genus. Namely, the total Weierstrass weight of all vertices is bounded by $g^3 - g$. We focus on graphs of genus 2 and show that there are no graphs of genus 2 where all the vertices are Weierstrass vertices and conjecture that there are no graphs of genus g with $g^3 - g$ vertices all of whose vertices are normal Weierstrass vertices.

Keywords: Arithmetic genus, divisor of a graph, gap sequence, Weierstrass vertex, Weierstrass weight