Oral presentation: 131

Controllability of a system of coupled harmonic oscillators

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In general, it is worthwhile to understand and control the dynamics of an existing system whose output behaves somewhat closer to the desired output rather than developing a new system which tracks the desired output, since it is beneficial for industries in many aspects; low cost, less time, etc. In this research, we control the output of a couple harmonic oscillator which has been extensively used in many Engineering Models by mainly focusing on two types of control techniques, namely source term controlling and initial condition controlling. Numerical results using MATLAB validates that controlled system output tracks the desired output for these two types of controlling. Consider the governing equations:

$$\left.\begin{array}{l} m\ddot{x}_1 = -\frac{mg}{l}x_1 + k(x_2 - x_1)\ , m\ddot{x}_2 = -\frac{mg}{l}x_2 + k(x_1 - x_2) \\ x_1(0) = \alpha, x_2(0) = \beta, \dot{x}_1(0) = \gamma, \dot{x}_2(0) = \mu. \end{array}\right\} \quad (1)$$

(i) Controlling by the source term: Let the desired outputs x_1 and x_2 be given by

$$x_1 = r\sin(pt) + w\sin(qt), x_2 = -r\sin(pt) + w\sin(qt)$$

where r, w, p, and q are parameters. Then controlled system for the source term is given by $m\ddot{x}_1 = -\frac{mg}{l}x_1 + k(x_2 - x_1) + \alpha(t)$, $m\ddot{x}_2 = -\frac{mg}{l}x_2 + k(x_1 - x_2) + \beta(t)$ and the source terms are $\alpha(t) = A\sin(pt) + B\sin(qt)$ and $\beta(t) = -A\sin(pt) + B\sin(qt)$, where

$A = -rmp^2 - sr + kr$, $B = -wmq^2 - kw - sw$ and $s = -\left(\frac{mg}{l} + k\right)$ for $x_1(0) = 0$, $x_2(0) = 0, \dot{x}_1(0) = \gamma, \dot{x}_2(0) = \mu$.

(ii) Controlling by the initial condition: Let the desired output x_1 and x_2 be given by

$$x_1 = ae^{\sqrt{\lambda+\phi}\,t} + be^{-\sqrt{\lambda+\phi}\,t} + ce^{\sqrt{\lambda-\phi}\,t} + de^{-\sqrt{\lambda-\phi}\,t},$$

$$x_2 = ae^{\sqrt{\lambda+\phi}\,t} + be^{-\sqrt{\lambda+\phi}\,t} - ce^{\sqrt{\lambda-\phi}\,t} - de^{-\sqrt{\lambda-\phi}\,t},$$

where $\lambda = -\left(\frac{g}{l} + \frac{k}{m}\right)$ and $\phi = \frac{k}{m}$. By considering the system (1) the controlled system for initial conditions is given by $m\ddot{x}_1 = -\frac{mg}{l}x_1 + k(x_2 - x_1)\ , m\ddot{x}_2 = -\frac{mg}{l}x_2 + k(x_1 - x_2)$ with initial conditions:

$x_1(0) = \alpha + (a + b - c - d) - (A + B - C - D), x_2(0) = \beta + (a + b + c + d) - (A + B + C + D), \dot{x}_1(0) = \gamma + \left(\frac{(a-b)-(A-B)}{\sqrt{\lambda+\phi}} + \frac{(C-D)-(c-d)}{\sqrt{\lambda-\phi}}\right), \dot{x}_2(0) = \mu + \left(\frac{(a-b)-(A-B)}{\sqrt{\lambda+\phi}} + \frac{(c-d)-(C-D)}{\sqrt{\lambda-\phi}}\right)$ where $A = \frac{1}{4}\left(\alpha + \beta + \frac{\gamma}{\sqrt{\lambda+\phi}} + \frac{\mu}{\sqrt{\lambda+\phi}}\right), B = \frac{1}{4}\left(\alpha + \beta - \frac{\gamma}{\sqrt{\lambda+\phi}} - \frac{\mu}{\sqrt{\lambda+\phi}}\right)$,

$$C = -\frac{1}{4}\left(\alpha - \beta + \frac{\gamma}{\sqrt{\lambda-\phi}} - \frac{\mu}{\sqrt{\lambda-\phi}}\right)\ , D = -\frac{1}{4}\left(\alpha - \beta - \frac{\gamma}{\sqrt{\lambda-\phi}} - \frac{\mu}{\sqrt{\lambda-\phi}}\right).$$

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