**Conference Paper No: MF-04**

# Applicability of modified queueing model with encouraged arrivals for economic recession

J.A.S. Dinushan* and C.K. Walgampaya

Department of Engineering Mathematics, University of Peradeniya, Sri Lanka
sanoj.j@eng.pdn.ac.lk*

## Abstract

Businesses often offer lucrative deals and discounts so that customers' encouragement to engage with those firms are developed. This kind of arrivals are termed as encouraged arrivals. As a pandemic is overwhelming the world, it could be suggested that, for upbringing of declining businesses, this concept, now, deserves to be taken into account more than ever in the past. In order to describe the encouraged arrival process mathematically, a Markovian queuing model is used and the parameter that represents the arrival rate is modified with percentage increase in the arrival rate of customers. In this paper, we investigate the behavior of measures of performances with and without encouraged arrivals for multi-server finite capacity queuing system. In the analysis it was possible to identify that a significant number of customers compared to the normal arrival process is engaged with the system when it is affiliated with the encouraged arrival process. As well as it shows that customers engage more and more with the system irrespective of the high rates of arrivals and low rates of services. In addition to that under the economic analysis, it was uncovered that the profit increment due to encouraged arrivals is very higher than that due to normal arrivals.

#### Keywords

Covid-19, Economic Recession, Encouraged Arrivals, Queuing System

## Introduction

The advancement of the cutting-edge technology and the rising concept of globalization has influenced businesses so that the environment of businesses gets more competitive than ever before. As well as customers have become more selective and the availability of different brands in each specific product has been resulted in brand switching more frequently. So, it has become a more decisive challenge for every business to maintain a more attractive environment for drawing the attention of as many customers as possible. As a business becomes more copious, the attraction of customers is involuntarily drawn to it and as more customers get engaged with the business, the development of business is ensured. In that case, customer management plays a more catalytic role to ensure a high degree of interest in customers. As well as in the contemporary situation of COVID 19 pandemic, businesses are one of the most affected areas. Varieties of solutions are suggested to overcome from this unpleasant condition and in this case also encouraged arrivals can be regarded as a more effective solution.

Considering customer management, queueing theory deserves to be paid special attention as on many occasions waiting lines are formed when the arrival rate of customers outpaces the service rate of firms. Erlang has initially introduced the concept of queuing theory in the context of telephone traffic engineering (Erlang, 1909). By now, that concept is made use in various fields.

In depth study of queuing theory, we were able to identify number of research carried out regarding different practical approaches such as queuing systems with limited and unlimited length of waiting lines (Jiang, Khattak, Hu, Zhu, & Yao, 2016), and queuing systems with one service point (Som & Seth, An M/M/1/N queuing system with encouraged arrivals, 2017). However, as one of the most practical applications of waiting lines, we decided to study the behavior of a queuing system having multiple service points.

In addition to that, the effects of applicable measures that are usually taken by businesses to increase the number of customers who visit the business were mainly investigated. The arrivals of customers under such stimulations such as lucrative deals and various offers are called encouraged arrivals. The effect of encouraged arrivals was introduced into the model by making a slight change to parameters. In this paper we compare the differences of such encourage arrivals and normal arrivals of multiple server - finite capacity (M/M/c/N) queuing system under steady state probabilities in different aspects such as average number of customers gathered in the queue, profit that could be earned and cost of making that profit.

**Methodology**

A model that describes (M/M/c/N) queuing system was created and results were plotted against ranges of arrival rates and service rates.

Assumptions of the model:

(i) The arrivals occur one by one in accordance with the Poisson process with parameter $\lambda(1+\eta)$, where '$\eta$' represents the percentage increase in the arrival rate of customers, calculated from past or observed data.
(ii) Service times are exponentially distributed with parameter '$\mu$'.
(iii) Customers are serviced in the order of their arrival.
(iv) There are 'c' servers through which the service is provided.
(v) The capacity of the system is finite, say 'N'.

A. Steady-State Probabilities

The probability of 'n' customers in the system (Som & Seth, An M/M/1/N queuing system with encouraged arrivals, 2017)

$$P_n = \{\frac{1}{n!} r^n P_0, \qquad 0 \le n < c \;\; \frac{1}{c^{n-c}\,c} r^n P_0, \qquad c \le n \le N$$

The probability of no customers in the queue

$$P_0 = \{\left[\sum_{n=0}^{c-1} \quad \frac{r^n}{n!} + \frac{r^c}{c!}(1-\rho^{N-c+1})\frac{c}{c-r}\right]^{-1},$$

$$\rho \neq 1 \left[\sum_{n=0}^{c-1} \quad \frac{r^c}{n!} + \frac{r^c}{c!}(N-c+1)\right]^{-1}, \qquad \rho = 1$$

B. Measures of performance

Based on the steady-state probabilities, formulae for measures of performances were obtained.

Average number of customers in the system

$$L_s = \sum_{n=0}^{N} nP_n = L_q + c - P_0 \sum_{n=0}^{c-1} \frac{(c-n)(\rho c)^n}{n!}$$

Average number of customers in the queue

$$L_q = \frac{P_0 r^c \rho}{c!\,(1-\rho)^2}\left[1 - \rho^{N-c+1} - (1-\rho)(N-c+1)\rho^{N-c}\right] \qquad where\ \rho \neq 1$$

Average waiting time in the system $W_s = \frac{L_s}{r\mu(1-P_N)}$

Average waiting time in the queue $W_q = \frac{L_q}{r\mu(1-P_N)}$

where,

$\rho = \frac{\lambda}{c\mu}$ and $r = \frac{\lambda}{\mu}$ for normal arrivals, and $\rho = \frac{\lambda(1+\eta)}{c\mu}$ and $r = \frac{\lambda(1+\eta)}{\mu}$ for encouraged arrivals.

For same measures, the difference between with and without encouraged arrivals, were investigated

C. Economic model

$C_s$ = Cost per service per unit time.

$C_h$ = Holding cost per unit per unit time.

$C_L$ = Cost associated with each lost unit per unit time.

$R$ = Revenue earned per unit per unit time.

Following economic parameters were calculated using steady-state probabilities and measures of performances. (Som & Seth, An M/M/2/N Queuing System with Encouraged Arrivals, Heterogenous Service and Retention of Impatient Customers, 2017).

Total Expected Cost $TEC = C_s\mu + C_h L_s + C_L\lambda(1+\eta)P_N$

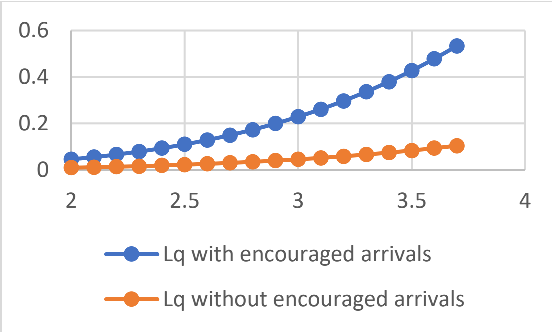
Total Expected Revenue $TER = R\mu(1-P_0)$

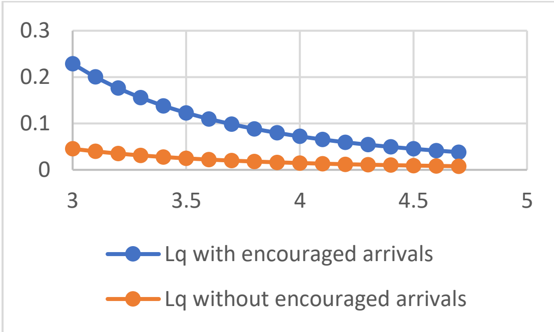
Total Expected Profit $TEP = TER - TEC$

Measures of performances, costs and profits were simulated for arrival rates ranging from 2 to 3.7 and service rates ranging from 3 to 4.7. Values for $C_s$, $C_h$, $C_L$ and $R$ were taken according to previous studies (Som & Seth, An M/M/1/N queuing system with encouraged arrivals, 2017). Simulation was carried out using Microsoft excel.

## Results

We have observed the variation in measures of performance with respect to arrival rate and service rate.

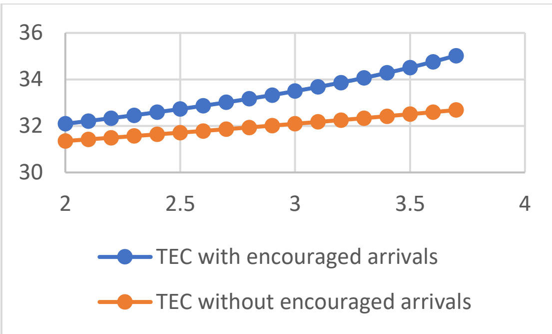


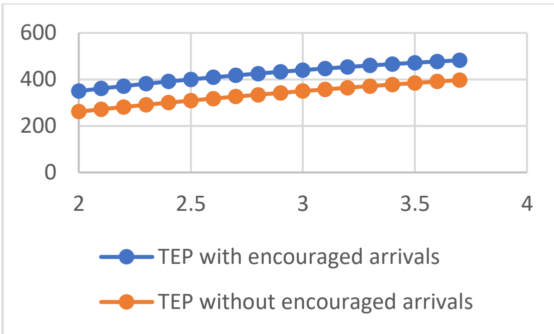




a. $Lq$ Vs $\lambda$ when $\mu = 3$ b. $Lq$ Vs $\mu$ when $\lambda = 3$

***Figure 1.*** *Simulation results when* $N = 10, c = 3, \eta = 0.5$

Then we observed the variation in economic parameters with respect to arrival rate.

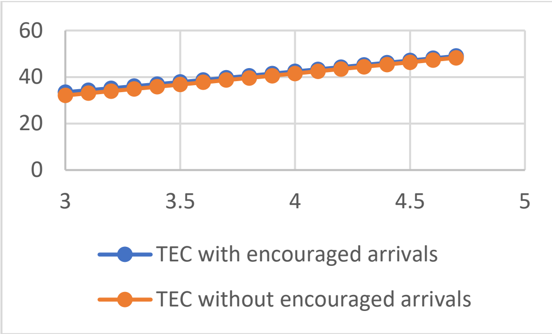


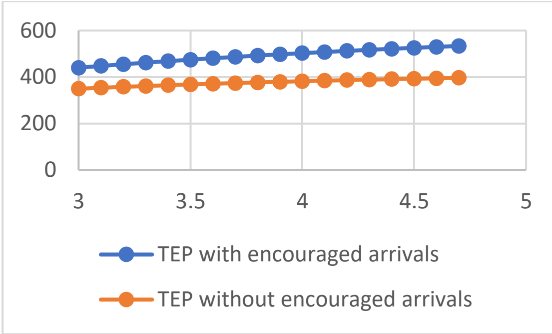

a. $TEC\ Vs\ \lambda$ b. $TEP\ Vs\ \lambda$

***Figure 2.*** *Simulation results when* $N = 10, c = 3, \eta = 0.5, \mu = 3, C_s = 10, C_L = 15, C_h = 2, R = 200$

Variation in economic parameters with respect to service rate is also observed.



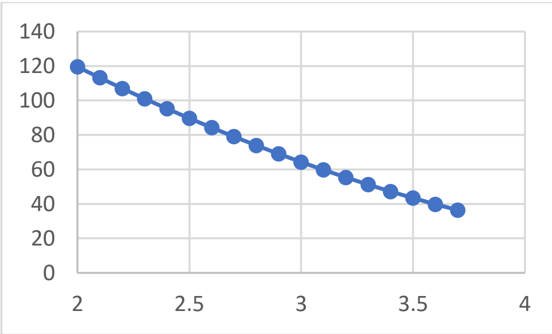

a. $TEC\ Vs\ \mu$



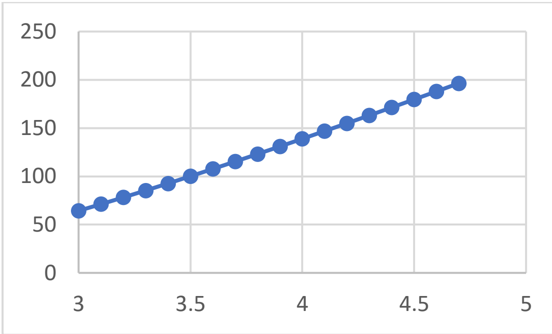

b. $TEP\ Vs\ \mu$

***Figure 3.*** *Simulation results for* $N = 10, c = 3, \eta = 0.5, \lambda = 3, C_s = 10, C_L = 15, C_h = 2, R = 200$

In addition to that the ratio between profit increment to cost increment due to the encouraged arrivals versus arrival rate and service rate are also investigated.




a. (Profit increment / Cost increment) Vs $\lambda$ when $\mu = 3$




b. (Profit increment / Cost increment) Vs $\mu$ when $\lambda = 3$

***Figure 4.*** *Simulation results for* $N = 10, c = 3, \eta = 0.5, C_s = 10, C_L = 15, C_h = 2, R = 200$

## Discussion

All the measures of performances always maintain a higher value when it is with encouraged arrivals. As the arrival rate increases, all the measures of performances increase, whereas as the service rate increases, all the measures of performances decrease. As well as it is conspicuous, there are significant differences between measures of performances (Lq) with and without encouraged arrivals in Figure 1.a and 1.b. Though the arrival rate becomes more and more high, the number of customers joining to the queue with an encouraged arrival process does not reduce; it becomes more higher compared to the number of customers joining to the normal arrival process (Figure 1.a). In a similar manner, it is possible to notice a very high number of customers in the encouraged arrival process compared to the normal process even at low service rates (Figure 1.b). The difference of these two situations can be considered as the willingness of customers to engage with a system with encouraged arrivals irrespective to the high arrival rates and low service rates.

Considering the cost analysis, both cost and profit always maintain a higher value for encouraged arrivals than normal arrivals. Yet the cost increased due to encouraged arrivals can be taken for granted compared to the profit increased due to encouraged arrivals. Having calculated according to the data taken from previous studies, it is possible to observe that the increment of TEP/increment of TEC due to encouraged arrivals is almost higher than 60 for many numbers of cases (considerable range of arrival rates and service rates) according to Figure 4.a and 4.b. As well as according to the Figure 3.a, the cost of encouraged and unencouraged arrivals almost remained undeferred, yet the profit shows a considerable increment due to encouraged arrivals. Furthermore, comparing Figures 2.a and 3.a, it is possible to witness that the cost increment due to encouraged arrivals as service rate changes is much lower than that as arrival rate changes. However, it is difficult to see a considerable change of differences between profits with and without encouraged arrivals when arrival or service rate changes as per Figure 2.b and 3.b.

**Conclusion**

More customers will retain in the system when the firm releases various offers and discounts so that it makes encouraged arrivals. That is drastically increased as the arrival rate increases and takes a considerable higher value though the service rate takes a low value. That represents the stimulation of customers when they see many numbers of customers are getting engaged with the firm and the interest due to the encouragement of them though they see the slowness of serving process. The profit that can be earned by encouraged arrivals is considerably larger compared to the amount of cost paid on it. As well as the profit is considerably developed by a slight increment of service cost. Considering all these aspects, it can be concluded that the concept of encouraged arrivals is preferable for upholding declining business and further developing undeveloped business. Furthermore, this concept could be taken into consideration to face successfully against the economic recession in this era due to COVID-19 pandemic.

**References**

Erlang, A. K. (1909). The theory of probabilities and telephone conversations. *Nyt Tidsskrift for Matematik, 20*, 33.

Jiang, Y., Khattak, A., Hu, L., Zhu, J., & Yao, Z. (2016). Analytical modeling of two-level urban rail transit station elevator system as phase-Type bulk service queuing system. *European Transport* (62), 5.

Som, B. K., & Seth, S. (2017). An M/M/1/N queuing system with encouraged arrivals. *Global Journal of Pure and Applied Mathematics, 13,* 3443-3453.

Som, B. K., & Seth, S. (2017). An M/M/2/N Queuing System with Encouraged Arrivals, Heterogenous Service and Retention of Impatient Customers. *Advanced Modeling and Optimization, 19*, 97-104.