Karunarathne, Sanjeewa & R.A.D.Piyadasa
Department of Mathematics, University of Kelaniya

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The inverse square potential and relativistic bound states

An important failure of the basic bag model is its failure to provide a pion-mediated interaction. We are mainly interested in investigating weather gross properties of the nucleon can be explained confining quarks by a potential other than three dimensional infinite potential well, infinitely deep spherical cavity. In this respect, we have investigated whether inverse square attractive potential can bind non-zero mass particle to a small region as in the case of MIT bag model[2].

We are motivated do so by the fact that the phase shift produced by a repulsive inverse square potential becomes complex when the net inverse square potential, attractive inverse square potential plus the centrifugal potential becomes negative, and the repulsive square well potential phase-shift is independent of the incident energy of a scattering particle[1] in case of non-relativistic quantum mechanics.

Solution of Dirac equation for inverse square attractive potential

Let us solve the Dirac equation for $V = -\frac{\beta}{r^2}$, where β is positive. In this respect we have to solve the coupled differential equations (1.1b) and (1,1c) in the following.

$$g' - g + \frac{kg}{\rho} - \left(\frac{\alpha_2}{\alpha} - \frac{\beta}{\hbar c \alpha r^2}\right) f = 0 \qquad (1.1a)$$

$$g' - g + \frac{kg}{\rho} - \left(\frac{\alpha_2}{\alpha} - \frac{\gamma}{\rho^2}\right) f = 0 \qquad (1.1b)$$

where $\gamma = \frac{\beta\alpha}{\hbar c}$.

$$f' - f - \frac{kf}{\rho} - \left(\frac{\alpha_1}{\alpha} + \frac{\gamma}{\rho^2}\right) g = 0 \qquad (1.1c)$$

α_1 and α_2 carry their usual meaning and we look for solutions of (1.1b) and (1.1c) in the form of power series;

$$f = \sum_{\nu=0}^{\infty} a_\nu \rho^{s+\nu} \qquad (1.2)$$

$$f' = \sum_{\nu=0}^{\infty} a_\nu (s+\nu) \rho^{s+\nu-1} \qquad (1.2a)$$

$$g = \sum_{\nu=0}^{\infty} b_\nu \rho^{s+\nu} \qquad (1.3)$$

$$g' = \sum_{\nu=0}^{\infty} b_\nu (s+\nu) \rho^{s+\nu-1} \qquad (1.3b)$$

It can be shown that

$$(sb_0 + kb_0 + \gamma a_1)\rho^{s-1} + \gamma a_0 \rho^{s-2} + \sum_{\nu=2}^{\infty}\left((s+\nu+k-1)b_{\nu-1} - b_{\nu-2}\frac{\alpha_2}{\alpha}a_{\upsilon-2} + \gamma a_\nu\right)\rho^{s+\nu-2} = 0$$

From this equation, it follows that $a_0 = 0$ and

$$sb_0 + kb_0 + \gamma a_1 = 0 \tag{1.4}$$

Again, we must have

$$\sum_{\nu=0}^{\infty} a_\nu (s+\nu)\rho^{s+\nu-1} - \sum_{\nu=0}^{\infty} a_\nu \rho^{s+\nu} - k\sum_{\nu=0}^{\infty} a_\nu \rho^{s+\nu-1} - \left(\frac{\alpha_1}{\alpha} + \frac{\gamma}{\rho^2}\right)\sum_{\nu=0}^{\infty} b_\nu \rho^{s+\nu} = 0 \tag{1.5}$$

$$(sa_0 - ka_0 - \gamma b_1)\rho^{s-1} - \gamma b_0 \rho^{s-2} + \sum_{\nu=2}^{\infty}\left((s+\nu-k-1)a_{\nu-1} - a_{\nu-2}\frac{\alpha_1}{\alpha} b_{\upsilon-2} - \gamma b_\nu\right)\rho^{s+\nu-2} = 0$$

Which gives $b_0 = 0$ and

$$sa_0 - ka_0 - \gamma b_1 = 0 \tag{1.6}$$

(1.4) and (1.6)) give $a_1 = b_1 = a_0 = b_0 = 0$. This implies that both g and f are identically zero.

Hence, we conclude that the inverse square potential cannot bind a relativistic particle. We conclude also that in relativistic scattering states there is no need to cut-off the potential tail as discussed in [3] and [4].

References

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