

KAUFFMAN BRACKET VERSUS JONES POLYNOMIAL SKEIN MODULES

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ABSTRACT. This paper resolves the problem of comparing the skein modules defined using the skein relations discovered by P. Melvin and R. Kirby that underlie the quantum group based Reshetikhin-Turaev model for $SU(2)$ Chern-Simons theory to the Kauffman bracket skein modules. Several applications and examples are presented.

1. MOTIVATION

In 1984 V.F.R. Jones introduced a polynomial invariant of knots and links [16]. Immediately after, L. Kauffman defined a similar polynomial knot and link invariant, the Kauffman bracket, which is in fact an invariant of framed knots and links [17]. Kauffman has shown how the Jones polynomial of a knot can be computed from the Kauffman bracket.

In 1989 E. Witten in [25] has explained the Jones polynomial by means of a quantum field theory based on the Chern-Simons functional. The Jones polynomial corresponds to the particular case of the Chern-Simons theory with gauge group $SU(2)$. By making use of physical intuition, Witten predicted the Jones polynomial to be part of a more general family of knot, link, and manifold invariants. Motivated by Witten's ideas, Reshetikhin and Turaev constructed the knot, link, and manifold invariants of the $SU(2)$ Chern-Simons theory using a quantum group associated to $sl(2, \mathbb{C})$ [23]. This theory fulfills Witten's predictions. An analogous theory was developed for the Kauffman bracket by Blanchet, Habegger, Masbaum, and Vogel in [1], and this theory parallels that of Reshetikhin and Turaev. Each of these two parallel theories have lead to significant developments and the aim of the present paper is to explain the relationship between the two models at the most general level.

Within the Reshetikhin-Turaev theory, and already present in previous works by Reshetikhin himself, lies the Jones polynomial of framed knots and links, but with a slightly different normalization. This polynomial fits exactly the quantum field theoretical model from Witten's paper, it is the polynomial that Chern-Simons theory would associate to a link whose components are colored by the 2-dimensional irreducible representation of $SU(2)$.

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