# Solve Manufacturer's Pallet Loading Problem (MPLP) with Practical Warehouse Constraints

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# Solve Manufacturer's Pallet Loading Problem (MPLP) with Practical Warehouse Constraints

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***Abstract*— This study presents a two-phase algorithm for solving the Manufacturer's Pallet Loading Problem (MPLP) while considering overhang and stability constraints using a block approach. The MPLP involves determining a loading pattern that can load the most identical rectangular boxes onto a larger rectangular pallet. The proposed model is developed to obtain the maximum number of boxes that could be loaded onto a layer considering and without considering the overhang according to eight predetermined block arrangements (BAs) in the first phase. Then, the obtained overhang solutions are checked against the stability constraint in the second phase. Finally, the maximum number of boxes per layer is determined based on the results from the two phases. The validity and the performance of the proposed algorithm have been tested on available datasets in the literature. The proposed algorithm increased the number of boxes per layer; one to six boxes while considering overhang and stability, improving the pallet area utilization by 2.5% up to 14.3% for the tested datasets.**

***Keywords—Manufacturer's Pallet Loading Problem (MPLP), overhang, stability, heuristic algorithm***

## I. INTRODUCTION

Pallet Loading Problem (PLP) is crucial in the logistics industry. The PLP is a Nondeterministic Polynomial-hard (NP-hard) problem [1] that involves determining a loading pattern that can load the maximum number of small rectangular items onto a large rectangular pallet. The PLP can be classified into two types based on the loading boxes dimensions. The first type is the Manufacturer's Pallet Loading Problem (MPLP): where items required for loading onto a pallet are homogeneous, whereas the second type is Distributor's Pallet Loading Problem (DPLP): where items required for loading onto a pallet are heterogeneous [1]. The objective of PLP is to load a maximum number of items onto a pallet while achieving maximum pallet utilization.

Regardless of category, several constraints consider when solving the PLP, such as fragility, geometry, and pallet strength. Overhang is one of the most critical and disregarded constraints when addressing the PLP. The overhang can be defined as an item hanging over the edge of the pallet or its base in pallet configuration. When this happens, the remaining two edges of an item that are not overhanging must do all the work for load support. Therefore, the overhang will be a cause decrease in the pallet strength, damage the pallet and load, and reduce the compression up to 30% [2]. But, due to the practical aspects and special situations, warehouses allow overhanging such as larger item dimensions, storing more items to reduce the required number of pallets, which is a cost. When an item is overhanging, the item stability must examine to increase the pallet stability and strength, which reduce damages to the pallets and items. Stability is the state of being stable. Loading pattern and stack height are the main factors that affect the stability of a pallet. When considering the stability of an item, at least a minimum percentage of more than 50% of an item base must be supported by the pallet base.

Overhang occurs primarily in the warehouses due to practical considerations. Typically, warehouses store homogeneous boxes within the same pallet rather than storing heterogeneous boxes due to the benefits, such as reducing the cycle count time and easy to identify and pick items. Warehouse activities account for 23% of logistics costs in the US and 39% in Europe [3]. A considerable portion of these costs is due to the overhang. Cause overhang is one of the primary reasons for pallet and item damages and reducing pallet strength. The dilemma is there is no way or model to consider overhang when loading the pallets. Therefore, a proper model that considers overhang with stability while loading a pallet will reduce the damages in pallet and items, increase the pallet strength while increasing the number of items for a pallet. The model will be beneficial to the manufacturers, warehouses, and businesses, which provide warehouse services such as 3PL companies by reducing the overall costs related to logistics management.

The remaining parts of this paper are as follows: Section II provides a review of relevant literature. Section III describes the proposed methodology in detail whereas. Section IV and section V illustrate the computational results, and conclusion respectively.

## II. LITERATURE REVIEW

Most research studies treated the Manufacturer's Pallet Loading Problem in two different ways. The first way is bi-dimensional (2D), and the second way is three-dimensional (3D). The difference between these two approaches is the bi-dimensional way considers the pallet and box area, while the three-dimensional way considers the pallet and box volume. Many research studies have studied the MPLP with 2D treatment [4], [5], [6], [7], and [8], and few studies have studied MPLP with 3D treatment [1], [9].

Studies have considered several constraints when solving the MPLP. Most research studies focused on maximizing the pallet utilization while considering non-overlap and non-overhang constraints [1], [6], [7], and [8]. Further, [9] has considered stability and geometric constraints and [1] has considered both maximum loading height and maximum loading weight of the pallet and dynamic compressive strength of the pallet.

Researchers have used different methods and approaches to solve the MPLP. [4] has proposed G4-heuristic, [5] has proposed L-approach, and [9] has proposed pinwheel pattern to solve the MPLP. [7] and [10] have used branch and bound algorithms to obtain the results, while [10] has used a staircase structure for the model building. Block approach has been used by [1], [8], and [11] for their models and [8] has taken mixed-integer linear programming to solve the model. Furthermore, both [6] and [1] studies have used two-phase algorithms in their models and [6] has used integer linear

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programming model. All the studies have shown improved performance according to their results.

There are many studies related to the MPLP that consider non-overhang as a constraint, and no work of literature found addressing the MPLP while considering overhang and stability constraints. However, the study [1] has mentioned that, their study can be extended in future to consider overhang. Therefore, this study focuses on solving the MPLP while considering overhang and stability constraints.

## III. Methodology

The manufacturer's pallet loading problem can be identified as an NP-hard problem. Hence, to obtain a near-optimal solution, heuristic methods are appropriate. When traditional approaches are too slow to solve a problem, or when traditional methods fail to provide a precise outcome, a heuristic is a strategy for finding an approximate solution. The primary drawback of the heuristic method is an optimal solution is not guaranteed for a problem in some circumstances.

Therefore, this paper presents a two-phase heuristic algorithm for solving the MPLP while considering overhang and stability constraints using a block approach. The number of boxes per horizontal layer considering overhang and without considering overhang according to eight possible predetermined block arrangements (BAs) are calculated separately at the first phase. Then, the obtained overhang solutions are checked against the stability constraint in the second phase. Finally, the maximum number of boxes per layer is determined based on the results obtained in both phases.

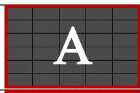
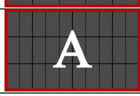
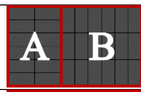
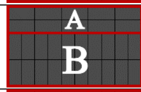
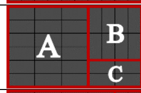
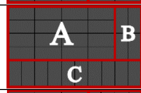
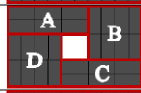
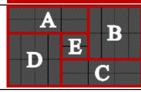
A block approach is used to arrange homogeneous boxes on the pallet layer. A pallet layer can have a maximum of five blocks according to eight possible predetermined block arrangements, and each block should be arranged uniformly in either an H-box or V-box orientation. If the length of the box is parallel to the pallet length, then the pattern is called the H-box, as shown in Fig. 1, and else, the box length is parallel to the width of the pallet, then the pattern is called the V-box, as shown in Fig2.

Fig. 1. H-box

Fig. 2. V-box

The eight possible predetermined block arrangements per layer are as follows. A layer can have a maximum of five blocks, and the blocks have indicated using A to E.

TABLE 1. Predetermined Block Arrangements

| Blocks | Block Arrangement Code (BAs) | Possible arrangement | Arrangement |
|---|---|---|---|
| One block | 1.1 | H box arrangement |  |
| | 1.2 | V box arrangement |  |
| Two blocks | 2.1 | Length arrangement |  |
| | 2.2 | Width arrangement |  |
| Three blocks | 3.1 | Length arrangement |  |
| | 3.2 | Width arrangement |  |
| Four blocks | 4.0 | Length and width arrangement |  |
| Five blocks | 5.0 | Length and width arrangement |  |

The assumptions associated with the proposed model are as follows.

- All the boxes and pallets used are assumed to be rectangular.
- All the boxes have the option to rotate in only $90^0$.
- All the boxes must be placed orthogonally.

The parameters of the proposed model are as follows.

TABLE II. Parameters of the Proposed Model

| Notation | Description |
|---|---|
| PL | Pallet length |
| PW | Pallet width |
| $l$ | Box length |
| w | Box width |
| $O_L$ | Length overhang |
| $O_W$ | Width overhang |

The decision variables are defined as follows.

TABLE III. Decision Variables of the Proposed Model

| Notation | Description |
|---|---|
| $L_i$ | Length of $i^{th}$ block |
| $W_i$ | Width of $i^{th}$ block |
| $Z_i$ | Number of boxes in $i^{th}$ block |
| H | H-box |
| V | V-box |
| $r_l$ | Pallet length remainder |
| $r_w$ | Pallet width remainder |
| x | Number of H boxes that can be placed on pallet length |
| y | Number of H boxes that can be placed on pallet width |
| a | Number of V boxes that can be placed on pallet length |
| b | Number of V boxes that can be placed on pallet width |

The data flow diagram for the proposed model is as follows.

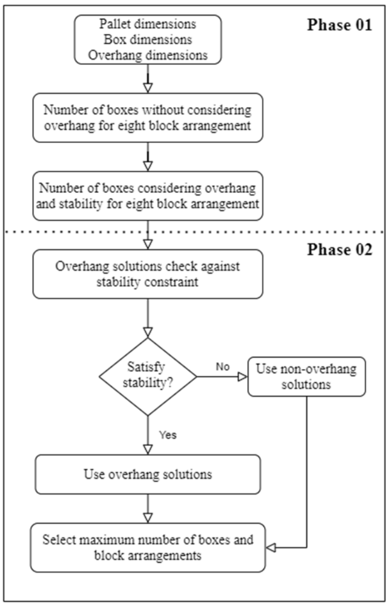



Fig. 3. Flow diagram for the proposed model

### *A. First Phase: Box Arrangement by Layer*

In this phase, the number of boxes per pallet layer is calculated considering overhang ($O_L > 0$ and $O_W > 0$) and without considering overhang ($O_L = O_W = 0$) according to the predetermined eight block arrangements separately.

Based on the given assumptions, parameters, and decision variables the model was developed and is as follows.

**Objective:**

Equation (1): To maximize the number of boxes that can be loaded onto a pallet considering overhang and stability.

$$\text{Max } Z = \sum_{i=1}^{5} (Z_i) \quad (1)$$

**Subject to:**

**Length orientation:**

Equation (2): The number of H and V boxes can be placed on the pallet length while reducing the pallet length remainder.

$$r_l \leq xH + aV \leq (PL + O_L)$$
$$(x, a \geq 0 \text{ and integer}, 0 \leq r_l < w) \quad (2)$$

**Width orientation:**

Equation (3): The number of H and V boxes can be placed on the pallet width while reducing the pallet width remainder.

$$r_w \leq yH + bV \leq (PW + O_W)$$
$$(y, b \geq 0 \text{ and integer}, 0 \leq r_w < w) \quad (3)$$

**Constraints:**

**Non-overlapping constraint:**

Equation (4) & (5): To avoid the overlapping of blocks.

$$(L_1 + L_3) \leq (PL + O_L) \text{ and } (W_1 + W_3) \leq (PW + O_W) \quad (4)$$

$$(L_2 + L_4) \leq (PL + O_L) \text{ and } (W_2 + W_4) \leq (PW + O_W) \quad (5)$$

The first step is to calculate the x and a values and y and b values while reducing the pallet length remainder and width remainder to find the number of H and V boxes that can be placed on pallet length and width respectively. In the second step the block dimensions calculate using the x, a, y, and b values according to the $(L_1, W_1) = (L_3, W_3) = (x*l, y*w)$, $(L_2, W_2) = (L_4, W_4) = (a*w, b*l)$ and $(L_5, W_5) = \{\text{positive}(L_1 - L_4)$ or $\text{positive}(L_3 - L_2)$, $\text{positive}(W_2 - W_1)$ or $\text{positive}(W_4 - W_3)\}$ based on block arrangements. As the third step, if there are no-overlapping blocks then the number of boxes per horizontal layer where $Z_i = \{(L_i/l)*(W_i/w)\}$ for (i = 1, 3), $Z_i = \{(L_i/w)*(W_i/l)\}$ for (i = 2, 4) and $Z_5 = \{[(L_5/l)*(W_5/w)]$ or $[(L_5/l)*(W_5/w)]\}$ is determine according to eight predetermined block arrangements. These calculations are done considering overhang ($O_L > 0$ and $O_W > 0$) and without considering overhang ($O_L = O_W = 0$) separately.

### *B. Second Phase: Checking Stability Constraint*

In this phase, the overhang results obtained for all eight predetermined box arrangements from the first phase were separately checked against the box stability constraint. When considering the stability of an item, at least a minimum percentage of more than 50% of an item base must be supported by the pallet base. Therefore, to have proper stability, this paper considered, at least 70% of the lower surface of the box must lie on the pallet base, that allows 30% of the box to overhang.

Equation (6) & (7): The length and width box stability.

$$\text{Length box stability } (S_L) = (0.3 * l) \quad (6)$$

$$\text{Width box stability } (S_W) = (0.3 * w) \quad (7)$$

**Length stability:**

Equation (8) & (9): The length overhang must be less than or equal to the length or width box stability percentage or both according to the box arrangement.

$$(L_1 + L_2) - PL \leq (S_L) \text{ or } (L_1 + L_2) - PL \leq (S_W) \quad (8)$$

$$(L_3 + L_4) - PL \leq (S_L) \text{ or } (L_3 + L_4) - PL \leq (S_W) \quad (9)$$

**Width stability:**

Equation (10) & (11): The width overhang must be less than or equal to the length or width box stability percentage or both according to the box arrangement.

$$(W_1 + W_4) - PW \leq (S_L) \text{ or } (W_1 + W_4) - PW \leq (S_W) \quad (10)$$

$$(W_2 + W_3) - PW \leq (S_L) \text{ or } (W_2 + W_3) - PW \leq (S_W) \quad (11)$$

**Upper bound ($Z_{UP}$):**

Equation (12): Maximum number of boxes can be loaded onto a pallet considering overhang.

$$z_{up} = \frac{(PL + O_L) * (PW + O_W)}{(l * w)} \quad (12)$$

The first step is to calculate the length and width overhang dimensions for an arrangement. Then the determined overhang dimensions are checked against the stability constraint of 70%: at least 70% of the lower surface of the box must lie on the pallet base, which allows 30% of the box to overhang as the second step. So, calculated length and width overhang dimensions must be less than or equal to the length or width box stability constraint as in equations (6) and (7) values or both according to the arrangement. If length and width overhang dimensions violate the stability constraint, then the calculated relevant non-overhang solution is the result of the layout. The third step is to select the maximum value and box arrangement as the solution from the calculated results. The designated maximum value must be less than or equal to the upper bound constraint value in equation (12): the maximum number of boxes that can load on to pallet layer considering overhang.

## IV. Computational Results

The proposed two-phase heuristic algorithm was coded in Python 3.9.6 programming language and tested on a 64-bit computer with an Intel i5-8250U (1.80GHz CPU, 4GB of memory) and a Windows 10 operating system.

### A. *Box Arrangement by Layer Without Considering Overhang Constraint*

This section compares the computational results obtained using frequent datasets by algorithms available in the studies with the proposed algorithm without considering overhang to validate the model, as shown in Table IV. Datasets (1-11) were solved using both exact algorithms and heuristic approaches, and datasets (12-15) were only solved using the heuristic approaches.

Table IV reports the results for the frequent instances without considering the overhang. Columns 2-6 in Table IV are the dimensions of the pallet PL and PW (in units), the individual box dimensions *l* and w (in units), the solutions obtained by the studies $Z_S$, the results obtained from proposed model *Z*, and the studied where the datasets were tested, respectively.

TABLE IV. The Computational Results Without Considering Overhang

| No. | (PL, PW) | (*l*, w) | $Z_S$ | Z | Studies |
|---|---|---|---|---|---|
| 1 | (46.9, 38.3) | (9.375, 4.812) | 40 | 40 | [1] |
| 2 | (48, 40) | (6.5, 3.438) | 82 | **83** | |
| 3 | (47.6, 39.4) | (6.125, 6.25) | 42 | 42 | |
| 4 | (48, 40) | (10.94, 8.375) | 18 | **19** | |
| 5 | (1000, 1000) | (200, 150) | 33 | 33 | [6], [12] |
| 6 | (22, 16) | (5, 3) | 23 | 23 | [6], [12] |
| 7 | (16, 11) | (3, 2) | 29 | 29 | |
| 8 | (50, 36) | (11, 7) | 23 | 23 | |
| 9 | (3750, 3063) | (646, 375) | 46 | 46 | |
| 10 | (34, 23) | (5, 4) | 38 | 38 | |
| 11 | (300, 200) | (21, 19) | 149 | 149 | [4], [13] |
| 12 | (32, 22) | (5, 4) | 34 | 34 | [14], [15] |
| 13 | (32, 27) | (5, 4) | 42 | 42 | |
| 14 | (40, 26) | (7, 4) | 36 | 36 | |
| 15 | (100, 64) | (17, 10) | 36 | 36 | |

Table IV indicates the obtained computational results of the proposed algorithm and past works of literature while not considering the overhang constraint. For all the datasets except datasets 3 and 5, the proposed algorithm has provided identical solutions. For datasets 3 and 5 of [1] study, the proposed algorithm has shown superior solutions. The proposed algorithm has provided 83 boxes per pallet layer while the study has shown 82 boxes for dataset 3. For dataset 5, the proposed algorithm has shown 19 boxes, while the literature has shown 18 boxes. So, for all the datasets tested, the proposed algorithm has provided improved or identical solutions compared to the studies.

### B. *Box Arrangement by Layer Considering Overhang and Stability Constraints*

This section shows the results obtained using frequent datasets considering overhang and stability constraints to evaluate the performances of the proposed algorithm. The allowable length and width overhang cannot be of large value due to practical considerations such as pallet and box dimensions, the distance between two pallet locations and vertical pallet stability. Therefore, $1 \leqslant O_L \leqslant 3$ and $1 \leqslant O_W \leqslant 3$ consider as the overhang dimensions for the following datasets.

The proposed two-phase heuristic algorithm provides the number of boxes considering overhang and stability for all eight predetermined block arrangements. Furthermore, the proposed algorithm delivers the overhang and block dimensions and the number of boxes for each block for all arrangements. There can be more than one block arrangement that provides the exact optimal solution. The block arrangement is depending on the pallet dimensions, box dimensions and overhang dimensions. Thus, the block arrangements which provide the optimal solution without considering overhang may differ from the block arrangements which provides the optimal solution considering overhang and stability.

Tables V and VI compare the results for dataset in Table IV under No. 10, where pallet length (PL) = 34, pallet length (PW) = 23, box length (*l*) = 5, box width (w) = 4, and the number of boxes without considering overhang is 38. Table V displays the number of boxes and block dimensions for all predetermined block arrangements without considering overhang, while Table VI displays the number of boxes and overhang and block dimensions for all block arrangements considering $O_L = O_W = 1$ overhang and stability.

The following tables show the predetermined block arrangements BAs, the number of boxes obtained by the proposed model without considering overhang Z, the number of boxes obtained by the proposed model considering overhang and stability $Z_{OH}$, and length and width overhang dimensions $O_L$ and $O_W$.

TABLE V. The Block Arrangements and Dimensions for Dataset in Table IV Under No. 10 Without Considering Overhang

| BAs | Z | Block Dimensions (in units) | | | | |
|---|---|---|---|---|---|---|
| | | A | B | C | D | E |
| 1.1 | 30 | (30,20) | _ | _ | _ | _ |
| 1.2 | 32 | (32,20) | _ | _ | _ | _ |
| 2.1 | 34 | (10,20) | (24,20) | _ | _ | _ |
| 2.2 | 36 | (30,8) | (32,15) | _ | _ | _ |
| 3.1 | 0 | (0,0) | (0,0) | (0,0) | _ | _ |
| 3.2 | 37 | (30,8) | (4,5) | (32,15) | _ | _ |
| 4.0 | 32 | (10,8) | (24,15) | (10,8) | (8,15) | _ |
| 5.0 | 38 | (10,8) | (24,15) | (10,8) | (8,15) | (16,8) |

Table VI displays the number of boxes and overhangs and block dimensions for all block arrangements considering

$O_L$=$O_W$ = 1 overhang and stability for the dataset in Table IV under No. 10.

TABLE VI. THE BLOCK ARRANGEMENTS AND DIMENSIONS FOR DATASET IN TABLE IV UNDER NO. 10 CONSIDERING OVERHANG AND STABILITY

| BAs | $Z_{OH}$ | ($O_L$, $O_W$) | Block Dimensions (in units) | | | | |
|---|---|---|---|---|---|---|---|
| | | | A | B | C | D | E |
| 1.1 | 42 | (1,1) | (35,24) | _ | _ | _ | _ |
| 1.2 | 32 | (0,0) | (32,20) | _ | _ | _ | _ |
| 2.1 | 38 | (1,1) | (15,24) | (20,20) | _ | _ | _ |
| 2.2 | 39 | (1,1) | (35,4) | (32,20) | _ | _ | _ |
| 3.1 | 42 | (1,1) | (15,24) | (20,20) | (20,4) | _ | _ |
| 3.2 | 0 | (0,0) | (0,0) | (0,0) | (0,0) | _ | _ |
| 4.0 | 38 | (1,1) | (15,4) | (20,20) | (15,4) | (12,20) | _ |
| 5.0 | 39 | (1,1) | (15,4) | (20,20) | (15,4) | (12,20) | (5,4) |

The optimal solution without considering the overhang is 38 boxes, as reported by Table V. Only one block arrangement provides the optimal solution, 5.0. According to Table VI, the optimal solution considering $O_L$ = $O_W$ = 1 overhang and stability is 42 boxes. Two block arrangements provide the identical optimal solution, 1.1 and 3.1. The proposed algorithm has increased the number of boxes per pallet layer by four boxes considering overhang and stability.

The proposed algorithm has increased the number of boxes by twelve, four, three, six and one when considering overhang and stability in block arrangements 1.1, 2.1, 2.2, 4.0 and 5.0, respectively. Table V shows solutions for seven out of eight predetermined block arrangements, while Table VI shows different seven out of eight predetermined block arrangements with different block dimensions. The reason for this is the arrangements that depend on the pallet, box, and overhang dimensions. Therefore, to obtain the optimal solution, all eight predetermined block arrangements must consider at the second phase of the algorithm.

Table VII compares the pallet area utilization (PAU) while considering overhang and stability and not considering overhang for the predetermined block arrangements. The proposed algorithm calculates the number of boxes per pallet layer. Thus, the pallet area utilization is used to evaluate the efficiency of the two-phase algorithm. Pallet area utilization is calculated using the following formula: used pallet area/ total pallet area. Columns 4-6 in Table VII are pallet area utilization without considering overhang PAU(Z), pallet area utilization considering overhang and stability PAU($Z_{OH}$), and the improvement in the pallet area utilization PAU(Imp).

TABLE VII. THE PALLET AREA UTILIZATION FOR BLOCK ARRANGEMENTS

| BAs | Z | $Z_{OH}$ | Pallet Area Utilization (%) | | |
|---|---|---|---|---|---|
| | | | PAU(Z) | PAU($Z_{OH}$) | PAU(Imp) |
| 1.1 | 30 | 42 | 76.7% | 100.0% | 23.3% |
| 1.2 | 32 | 32 | 81.8% | 81.8% | 0.0% |
| 2.1 | 34 | 38 | 87.0% | 97.2% | 10.2% |
| 2.2 | 36 | 39 | 92.1% | 99.7% | 7.6% |
| 3.1 | 0 | 42 | 0.0% | 100.0% | 100.0% |
| 3.2 | 37 | 0 | 94.6% | 0.0% | -94.6% |
| 4.0 | 32 | 38 | 81.8% | 97.2% | 15.4% |
| 5.0 | 38 | 39 | 97.2% | 99.7% | 2.5% |

Table VII indicates the obtained results for the pallet area utilization comparison. The maximum pallet area utilization is 100.0% by the 1.1 and 3.1 arrangements and the minimum improvement in pallet area utilization is 23.3%, shown by the 1.1 block arrangement.

Furthermore, the proposed algorithm provides the number of boxes per block for all predetermined block arrangements, as shown in Table VIII.

TABLE VIII. THE ALLOCATED NUMBER OF BOXES FOR EACH BLOCK CONSIDERING BLOCK ARRANGEMENT, OVERHANG AND STABILITY

| BAs | $Z_{OH}$ | Allocated Number of Boxes per Block | | | | |
|---|---|---|---|---|---|---|
| | | A | B | C | D | E |
| 1.1 | 42 | 42 | _ | _ | _ | _ |
| 1.2 | 32 | 32 | _ | _ | _ | _ |
| 2.1 | 38 | 18 | 20 | _ | _ | _ |
| 2.2 | 39 | 7 | 32 | _ | _ | _ |
| 3.1 | 42 | 18 | 20 | 4 | _ | _ |
| 3.2 | 0 | 0 | 0 | 0 | _ | _ |
| 4.0 | 38 | 3 | 20 | 3 | 12 | _ |
| 5.0 | 39 | 3 | 20 | 3 | 12 | 1 |

Table IX reports the results for selected frequent datasets considering overhang and stability from Table IV. Column 5 in Table IX is the improvement in the pallet area utilization PAU(Imp) compared to the without considering overhang.

TABLE IX. THE COMPUTATIONAL RESULTS CONSIDERING OVERHANG AND STABILITY

| (PL, PW) | ($l$, w) | ($O_L$, $O_W$) | $Z_{OH}$ | PAU(Imp) | BAs |
|---|---|---|---|---|---|
| (46.9, 38.3) | (9.375, 4.812) | (0, 0) | 39 | 0.0% | 2.1 |
| | | (1, 1) | 40 | 2.5% | 1.1, 2.4 |
| | | (2, 2) | 40 | | |
| | | (3, 3) | 40 | | |
| (47.9, 39.4) | (6.25, 6.125) | (0, 0) | 42 | 0.0% | 1.1, 1.2, 2.1 |
| | | (1, 1) | 42 | | |
| | | (2, 2) | 48 | 12.2% | 1.2 |
| | | (3, 3) | 48 | | |
| (48, 40) | (10.94, 8.375) | (0, 0) | 19 | 0.0% | 5.0 |
| | | (1, 1) | 19 | | |
| | | (2, 2) | 22 | 14.3% | 3.1 |
| | | (3, 3) | 22 | | |
| (100, 64) | (17, 10) | (0, 0) | 36 | 0.0% | 3.1, 4.0 |
| | | (1, 1) | 38 | 5.3% | 4.0 |
| | | (2, 2) | 38 | | 2.2 |
| | | (3, 3) | 40 | 10.6% | 2.2 |
| (40, 26) | (7, 4) | (0, 0) | 36 | 0.0% | 3.2, 4.0 |
| | | (1, 1) | 37 | 2.7% | 3.2 |
| | | (2, 2) | 40 | 10.8% | 1.2 |
| | | (3, 3) | 40 | | |

Table IX represent the obtained solutions for frequent datasets from Table IV considering $1 \leqslant O_L \leqslant 3$ and $1 \leqslant O_W \leqslant 3$ overhang and stability. The proposed algorithm increased the number of boxes for a pallet layer by one to six boxes for the presented datasets while improving the pallet area utilization by 2.5% to 14.3%, while providing the block arrangements.

The proposed two-phase heuristic algorithm tested with frequent datasets available in [1], [4], [6], [12], [13], [14], [15] and [16] studies to validate the model in the first stage. The proposed algorithm has shown identical or superior solutions for the tested datasets compared to the studies. A selected instance was used to exhibit the functionalities of the proposed algorithm in the second stage. The proposed algorithm provided each block and overhang dimensions, the number of boxes per block and the total number of boxes per layer according to the eight block arrangements separately. The optimal solution was selected among the eight results for the tested instance. The pallet area utilization was calculated to exhibit the performances of the proposed algorithm. The proposed algorithm increased the pallet area utilization up to 100% while providing the optimal solution for the presented instance in two block arrangements. Also, the proposed algorithm has been tested with several instances in studies considering $1 \leqslant O_L \leqslant 3$ and $1 \leqslant O_W \leqslant 3$ overhangs to exhibit its applicability. For the tested instances, the proposed algorithm increased the number of boxes from one to six while improving the pallet area utilization from 2.5% to 14.3% while providing the block arrangements with block dimensions and the number of boxes for each block. Most importantly, all the solutions were obtained within one second, which is a very reasonable computational time.

## V. Conclusion

In this study, a two-phase heuristic algorithm for solving the manufacturer's pallet loading problem (MPLP) is proposed while considering overhang and stability using a block approach. In the first phase, a mathematical model to determine the number of boxes loaded onto a pallet layer is developed while considering the overhang and not considering the overhang based on the eight predetermined blocks arrangements. The obtained overhang solutions in the first phase are checked against the 70% stability constraint in the second phase for all the block arrangements. When considering the stability of an item, at least a minimum percentage of more than 50% of an item base must be supported by the pallet base. Therefore, in order to have proper stability, in this paper it is considered that, at least 70% of the lower surface of the box must lie on the pallet base, which allows 30% of the box to overhang. Finally, select the maximum number of boxes per horizontal layer considering overhang and stability and block arrangement. We compared the proposed algorithm with frequent datasets available in studies to validate the model in the first stage. The proposed two-phase heuristic algorithm has shown identical or superior solutions for the tested datasets. Then, the proposed algorithm was tested with datasets in studies considering $1 \leq O_L \leq 3$ and $1 \leq O_W \leq 3$ overhangs and 70% stability constraint. The proposed algorithm has increased the number of boxes per layer by 1 to 6 boxes while considering overhang and stability, which improved the pallet area utilization by 2.5 % up to 14.3 %. The proposed algorithm will be applicable to the pallet configuration process in the warehouses and the manufacturing facilities. The algorithm will aid in deciding the number of boxes that can load onto a pallet while considering overhang in practical situations within a reasonable time. Also, the proposed algorithm will aid to increase the stability of the load and pallet while reducing the drawbacks of the overhang. We used the five-block heuristic approach, which divides the pallet base into at most five blocks to develop the proposed algorithm and in some circumstances, it was not sufficient to obtain the optimal solution. Therefore, in future, it can be used the recursive five-block approach to overcome such situations. Also, the study can be extended to address the distributor's pallet loading problem (DPLP) and additional constraints in future.

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