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Stochastic modelling of Lotka-Volterra competition

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The concept of stochasticity, which is based on probability theory, has played a vital role in describing the population fluctuations in most species. Demographic and environmental stochasticity are the main branches of stochasticity and occur due to the random nature of events and irregular or noisy dynamics, respectively. In particular, this study focuses on the concept of demographic stochasticity for studying the distribution of two competing populations. In the literature, the deterministic models of two competing populations have been studied, including the Lotka-Volterra competition model. Unlike prior work, we analyse the stochastic modelling of two competing populations where one population is subject to the Allee effect and understocking. The deterministic model of two competing populations, which is based upon the classical Lotka-Volterra competition model, is used to construct the corresponding continuous-time Markov chain (CTMC) and Ito stochastic differential equations (SDEs). Moreover, in the construction of CTMC and SDEs, demographic variability due to random birth and death have been applied to the populations, which is absent in the deterministic setting. In addition, the moments of the random variables in the populations based on the moment-generating functions of the transition probabilities are derived theoretically in such a way that the transition probabilities satisfy the forward Kolmogorov differential equations. Also, there is an infinite number of SDE models that correspond to the same ordinary differential equation system. In this study, we formulate two SDE models considering two different birth and death rates to see the variability in population interactions. The parameter values are taken from existing literature to justify the analytical results. The Euler-Maruyama numerical method is applied to simulate the numerical solutions of the Ito stochastic differential equations for comparing both types of stochastic models with the deterministic system numerically. From the numerical simulation, we have observed that the sample paths of the SDEs are closer to the solution of the deterministic model. Moreover, variabilities of the population interactions are highly correlated with the birth and death rates. In addition, for the chosen parameter values, though the populations coexist in the deterministic setting, we capture sudden population extinction in the stochastic setting. The study concludes that the theoretical results established in the deterministic setting may not be valid in the stochastic models due to random effects of the birth and death process embedded in the populations. Therefore, stochastic modelling with the Allee effect and stocking can significantly affect the competition outcomes and population interactions.

Keywords: Demographic stochasticity, Ito stochastic differential equations, Stocking