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Value at Risk Estimation for the BRICS Countries: A Comparative Study

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Abstract

This paper aims to investigate some statistical methods to estimate the value-at-Risk (VaR) for stock returns in the BRICS countries from 2011 to 2018. Four different risk methods are used to estimate VaR: Historical Simulation (HS), Risk metrics, Historical Method and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Process. By applying the Backtesting technique, we test the effectiveness of these different methods by comparing the calculated VaR with the actual realized losses (or gains) of the portfolio or the index. The results show that for the all-BRICS countries and at different confidence levels, the Historical Method and the Historical Simulation are the appropriate methods, while the GARCH model failed to predict precisely the VaR for all BRICS countries.

Keywords: Backtesting, BRICS, Confidence level, GARCH, Historical Method, Historical Simulation, Risk metrics, Value-at-Risk

JEL Classification: C01; C58; D84; G00; G17

1. Introduction

The quantification, forecasting and management of market risks are major concerns for financial institutions. This is because exposure to extreme price fluctuations in financial markets can lead to sudden and significant losses. Therefore, managers and researchers are responsible for ensuring financial stability.

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So, they should rely on an extensive database and metrics tables to correctly identify potential risks.

In recent years, many concepts of risk measurement have been developed. The main risk management methodology is the Value-at-Risk VaR method, which combines with other risk minimization techniques to achieve optimal results. VaR is the largest portfolio loss we can expect over a given period with a certain level of confidence. This value is a simple and easily understandable number which presents the risk to which the institution is exposed in the financial market. Despite its simple implementation, VaR has been the subject of several criticisms (Artzner et al., 1999, Yamai et Yoshiba, 2002, 2005, Sobreira et al. 2020).

In this research, we will compare the performance of different VaR estimation techniques for the BRICS countries (Brazil, Russia, India, China, and South Africa) from 2011 to 2018. We underline that this Research compares VaR based on the stock returns of the market indexes. Choosing an appropriate measure of VaR that gives an accurate estimate is an essential but a challenging task. In this study, VaR is estimated using four different risk methods: Historical Simulation (HS), Risk Metrics, Historical Method and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) process.

Our objective through this research is to improve the existing literature that deals with risk management by measuring VaR. Indeed, many research studies have studied the performance of these different methods, particularly in the context of BRICS countries. However, the goal of this research is to test the reliability of the different methods to retain the best methods which estimate the VaR. VaR's results will be evaluated with backtesting and compared using a loss function approach.

Based on the previously mentioned objectives, the problem that can be outlined is: What is the most reliable method for estimating VaR and to what extent do changes in data and confidence level influence performance and reliability value-at-risk (VaR) measures in BRICS countries?

2. VaR Estimation Method

VaR is a measure of the risk of loss of investments. It estimates how much a set of investments could lose (with a given probability), under some market conditions, over a defined period such as a day. Businesses and financial sector regulators use VaR to assess the amount of assets needed to cover potential losses (Bonga-Bonga and Nleya, 2016). This definition accepted by all financial investors is as follows: "VaR is the maximum potential loss that a portfolio can suffer, for a given time horizon and a given level of probability, assuming that this portfolio remains unchanged for the specified horizon". Manganelli and al. (2001)

Bayer (2018) argues that although it is difficult, it is crucial to choose between alternative modelling and value-at-risk (VaR) forecasting strategies. An improperly selected risk model can have dramatic effects on portfolios and the market, as evidenced by the stock market crash of 2015 when many standard approaches predicted insufficiently low levels of risk.

Choosing an appropriate VaR estimation method is an important but difficult task. Indeed, Hendricks (1996) suggested that further research aimed at comparing and combining the best features of the approaches examined might be useful. For this, it seems necessary to us to compare the different estimation methods of VaR, namely Risk Metrics, Historical Simulation, Historical Method, and Variance-Covariance Method, under the GARCH name.

2.1 Historical Simulation

Some researchers, such as Jawwad and Palgrave (2014), explain that Historical Simulation (HS) is the most popular and efficient method. The characteristics of the HS method:

- Relatively simple to set up
- Does not assume any form of distribution.
- Depends on the quality and availability of data.

According to Gajadharsingh (2013): "The empirical quantile method (or Historical Simulation) is a straightforward method of estimating risk measures. It is based on the empirical distribution of historical data on the returns of a financial portfolio. Formally, VaR is estimated simply by directly reading the empirical studies of past returns".

Wiener (1999) asserts that historical simulation belongs to the nonparametric method of calculating VaR. What is common to all nonparametric approaches is the empirical distribution, obtained from the observed data, as opposed to the parametric approach (where assumptions about the theoretical distributions of return are used). The main feature of historical simulation is its ease of implementation.

The Historical Simulation allows us to estimate the VaR of a portfolio by considering the amount invested in the portfolio in general and in each of its securities.

2.2 Historical Method

After identifying the significant risk factors for a financial market, we use the historical data collected to deduce the amount of loss. According to Didier (2014): "The historical method requires knowing the price history for an index to calculate the change in its value over time. This method is very inexpensive in terms of calculation and technique. In addition, no prior assumption on the form of the distribution is required. "

This method can determine a market index's daily Profits and Losses (P&L) which is then ranked in ascending order. Depending on the number of P&L calculated and the desired confidence interval, the historical VaR equals to the corresponding P&L value.

This simplicity of implementation generates many limits. While among its drawbacks, this method is unsuitable for derivative products (options, warrants, futures contracts, etc.). In addition, historical data must be sufficiently and widely large compared to the horizon of the VaR and its confidence level but not too much to ensure that the law of probability has not changed too much over the given period.

2.3 Risk Metrics

Risk Metrics was introduced in 1994. It contains datasets and techniques used to calculate the value at risk (VaR) of a portfolio of stocks or a market index. Morgan and Reuters collaborated in 1996 to develop the methodology and make the data widely available to practitioners, managers, and researchers. The objective is to improve and promote the transparency of market risks and subsequently create a benchmark for risk measurement by providing advice to clients on managing market risks.

Morgan calculates the VaR as the conditional variance as a weighted average shifted by one period and squared logarithms at period t-1 (Sobreira & Louro 2020):

$$\boldsymbol{\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2} \quad (1)$$

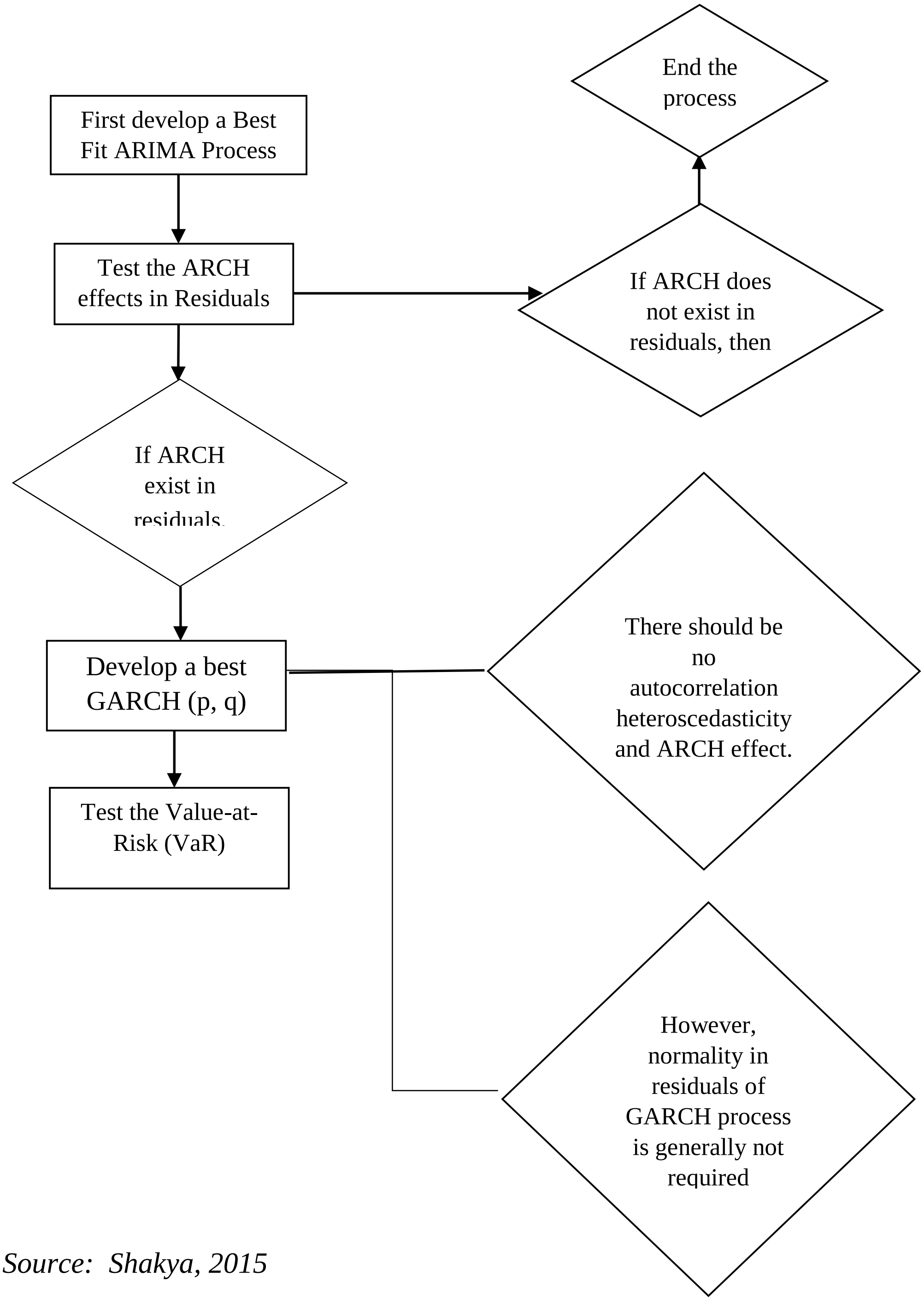
$\boldsymbol{\sigma_t^2}$**:** The conditional variance; $\boldsymbol{r_t^2}$**:** Square yield

Usually, λ = 0.94 for the forecast of daily volatility.

2.4 Generalized autoregressive conditional heteroscedasticity process (GARCH)

In 1982, Engle presented in his famous research paper entitled "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation" the family of conditional autoregressive heteroskedastic (ARCH) models. Since then, other research has focused on the modelling of conditional volatility, such as the work of Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994) and Diebold and Lopez (1995). Other papers have compared specific models for predicting conditional volatility, such as West and Cho (1994) and Heynen and Kat (1993).

Figure 1: The steps for calculating the VaR by the GARCH method



Source: Shakya, 2015

According to Angelidisa et al. (2004), the GARCH (p, q) model successfully captures several features of financial time series, such as thick-tailed returns and volatility clustering, as noted by Mandelbrot (1963) "… big changes tend to be followed by big changes in one or the other of the signs, and small changes tend to be followed by small changes…". On the other hand, the GARCH structure presents some drawbacks of implementation since the variance depends only on the magnitude and not on the sign ε_t, which is in contradiction with the empirical behavior of stock prices where a leverage effect may be present. This term, introduced by Black (1976), refers to the tendency of changes in stock returns to be negatively correlated with changes in return volatility, so that volatility tends to increase in response to bad news, (ε_t <0) and decrease in response to good news (ε_t > 0). Additionally, Brooks and Persand (2003) state that a VaR model that ignores asymmetries in the volatility specification is most likely to generate inaccurate predictions.

3. Backtesting

3.1 Definition

Considering the existence and the great diversity of methods for providing the VaR's estimation, many studies propose that different models applied for the same research generally led to very different estimates of VaR and therefore the risk for the same portfolio or the same market index.

Risk Managers need to assess VaR forecasts outside the regulatory standards imposed by Basel II by setting up Backtesting procedures (Silver et al., 2020).

Backtesting is a set of statistical procedures used in financial institutions to designate the testing of a strategy of a predictive model from existing historical data to verify that the actual losses observed are in line with the expected losses. This involves systematically comparing the historical VaR forecasts with the observed returns of the portfolio (Jorion, 2007).

This kind of simulation makes it possible to refine a model and verify hypotheses. Backtesting requires real historical data. According to Niepolla (2009): "The results of the Backtests provide an indication of potential problems within the system. A severe underestimation of risk is discovered, especially for stocks and stock options. However, the turbulent market environment poses challenges in evaluating backtesting results, as VaR models are only known to be accurate under normal market conditions".

However, such a Backtest involves some verification risks. First, we know that data from the past is not necessarily a guide to future performance. It is therefore desirable to keep realistic and simple assumptions. Over-optimizing a backtest would not lead to optimizing a strategy but to optimizing the past so that the strategy is always the best.

Backtesting must therefore make it possible to determine the most appropriate method (or methods) (Historical Simulation, Historical Method, Risk Metrics, GARCH) to predict the VaR. We must distinguish between the forecast validation test and the comparison test of forecasts, such as the Kupiec TUFF test, the Kupiec POF test or the Christoffersen independence test.

Only large institutions and professional fund managers use Backtesting because of the expense of obtaining and using detailed data sets. However, backtrading is used on a large basis and independent backtesting platforms. Although the technique is widely used, it has weaknesses.

3.2 Backtesting Value at Risk Forecast: Kupiec Pof-Test

The POF (proportion of failure) test examines whether the number of exceptions meets the given confidence level. The null hypothesis of failure is expressed as follows:

H0: $p = \hat{p} = \frac{x}{T} * 100$ (1)

Where:

p: Percentage of failure

$\hat{p}$: The observed failure rates

x: Number of exceptions

T: Number of total observations

Once the one-day VaR and the number of exceptions for each confidence level are known, the likelihood ratio test must be calculated.

If the calculated LR exceeds the critical value, the null hypothesis and the accuracy of the model must be rejected for a certain level of confidence.

The “LR” likelihood ratio test is expressed according to the following expression:

$$\boldsymbol{LR\ POF} = -2\ln\frac{((1-p)^{T-x} * p^{x})}{\left[1-(\frac{x}{T})\right]^{T-x} * \left(\frac{x}{T}\right)^{x}} \tag{2}$$

Where:

p: Confidence level

T: Total number of observations

x: Number of exceptions

4. Empirical Analysis

When we seek to invest in the stock market, we tend to focus on the developed markets of the European Union or the United States and we forget the emerging countries, namely the countries of the BRICS group, which are distinguished by their vast growing economies. Indeed, the BRICS countries attract a large part of capital inflows and represent a destination of choice for the investments of many global portfolio managers. The main problem with stock markets in developing countries is the access to markets and financial information. Unless investors know emerging markets like the back of their hand, they are therefore discouraged from investing in BRICS markets individually. They should therefore ask for funds and apply risk measurement methods based on historical data to build a complete idea of the market.

4.1 Empirical Results

4.1.1 Historical Simulation

The data used for the statistical calculations come from a secondary source, specifically, the share prices of 20 companies with the largest market capitalizations (see Annex 1) for a period of 2085 days. The data was collected via the “Datastream” financial and macroeconomic data platform. First, we assumed we have $ 2000 to invest in a portfolio at the rate of $ 100 for each company. Thus, daily returns are calculated for each company and then the daily return of the portfolio is calculated so that the daily returns of twenty companies are added up.

The third step is to calculate the overnight VaR for the portfolio at the confidence levels of 95% and 99%, respectively using the formula (percentile) in excel:

$$Var\ (99\%) = \ CENTILE\ (n1{:}\, nt;\ 99\%) \tag{3}$$

The daily losses are then considered to compare these values with the estimated calculation of the VaR. If the value of the portfolio loss is greater than the predicted overnight VaR value, then the exception exists. This comparison is necessary to see how many exceptions occur at the 95% and 99% confidence levels. (Annex 2)

Table 1: VaR estimation by the HS method (historical simulation)

Countries	Confidence level	VaR%	Number of exceptions	Number of total observations
Brazil	95% (α=5%)	-46,5514749	103	2085
	99% (α=1%)	-74,7337198	19	2085
Russia	95% (α=5%)	-38,10863374	109	2085
	99% (α=1%)	-61,97126617	32	2085
India	95% (α=5%)	-32,8803826	94	2085
	99% (α=1%)	-49,2278663	23	2085
China	95% (α=5%)	-44,29199755	95	2085
	99% (α=1%)	-74,22332947	29	2085
South Africa	95% (α=5%)	-35,38409499	122	2085
	99% (α=1%)	-61,00736923	20	2085

Source: Survey Data, 2022

The table shows the estimated VaR for the BRICS group at the 99% and 95% thresholds, as well as the number of exceptions (losses that have exceeded the VaR) and the number of total observations.

The highest number of exceptions is recorded in South Africa at the 95%. This means that following the estimation of the VaR by the HS, 122 values exceeded the worst expected loss in Brazil at the threshold by 95%, while there are only 20 losses that have exceeded the VaR at 99% threshold.

Generally, and depending on the results obtained, the VaR estimated at 99% is lower than that at 95% since the confidence level will be more limited (there is only a 1% chance that the losses will exceed the value at risk). And even for the number of exceptions (losses that exceeded VaR at 99% are therefore less than that at 95%).

4.1.2 The Historical Method

We carry out our analysis based on the repatriation of the daily closing values over the last 8 years (from 2011 to 2018) of the BRICS group market indices (BOVESPA, RTS, SENSEX, SSE, JSE). The data was extracted from the financial data platform "FactSet". We thus calculate the daily earnings, which are sorted by increasing value. The confidence level is then calculated from the number of observations (number of the day) according to the following expressions:

$$\text{VaR reference} = (\text{N° line} / \text{Total number of lines}) \tag{4}$$

$$\text{Var at } x\% = (100 - \text{Ref VaR})$$

The risk value is then obtained at the 99% and 95% levels by calculating the sorted gains. (Annex 3)

The daily losses are then considered to compare these values with the estimated calculation of the VaR. If the negative return (loss) of the index is greater than the expected overnight VaR value, the exception exists. This comparison is necessary to see how many exceptions occur at 95% and 99% confidence levels.

Table 2 presents the estimated VaR for each country of the BRICS group at the 99% and 95% thresholds. The number of exceptions (losses that exceed the VaR) and the number of total observations are different from each country due to national holidays and missing data for some indices.

The VaR for Brazil at 95% is equal to -1317 and -2017 at 99%, meaning that there is a 5% chance that the loss will exceed -1317 and a 1% chance that the loss will exceed - 2017.

The number of exceptions in the five countries is almost equal. On average, 19 Return (loss) values exceed the VaR at 99% for all countries. One hundred stocks were the exception at the level 95% threshold. This means that the measure of VaR by the Historical Method is robust and gives the exact estimates for all countries (it is no longer affected by the database).

Table 2: VaR estimation by the Historical method

Countries	Confidence level	VaR	VaR en % =	Number of exceptions	Number of total observations
Brazil	95% (α=5%)	-1317	1.62	98	1977
	99% (α=1%)	-2017	2.25%	19	1977
Russia	95% (α=5%)	-32,93	3.11%	99	2000
	99% (α=1%)	-62,87	5.44%	19	2000
India	95% (α=5%)	-351,28	0.94%	98	1972
	99% (α=1%)	-590,05	1.62%	19	1972
China	95% (α=5%)	-58,167	2.11%	96	1945
	99% (α=1%)	-175,56	6.58%	18	1945
South Africa	95% (α=5%)	-680,96	1.32%	103	2085
	99% (α=1%)	-1161,52	2.52%	20	2085

Source: Survey Data, 2022

4.1.3 Risk Metrics

As the "Historical Method" technique, we use the same database to calculate the daily returns of the market index for each country according to the following expression:

$$R_t = \ln(Pc_t / Pc_{t-1}) \qquad (5)$$

Where:

R_t: Daily returns

Pc_t: The closing price at time t.

Pc_{t-1}: The closing price at time t-1.

We then calculate the variance and the standard deviation to estimate the VaR at levels 95% and 99% by the following expression (See ***Annex*** 4):

$$VaR\,(1-\alpha) = \sigma_t * NORMAL.STANDARD.INVERSE.LAW.N(\alpha) \qquad (6)$$

Table 3: VaR estimation by risk metrics

Countries	Confidence level	VaR	Number of exceptions	Number of total observations
Brazil	95% (α=5%)	-2,388%	92	1977
	99% (α=1%)	-3,378%	20	1977
Russia	95% (α=5%)	-2,956%	83	2000
	99% (α=1%)	-4,181%	32	2000
India	95% (α=5%)	-1,576%	98	1972
	99% (α=1%)	-2,229%	30	1972
China	95% (α=5%)	-2,268%	82	1945
	99% (α=1%)	-3,208%	40	1945
South Africa	95% (α=5%)	-1,680%	103	2085
	99% (α=1%)	-2,376%	36	2085

Source: Survey Data, 2022

Table 3 shows the estimated VaRs for each country of the BRICS group at the levels of 99% and 95%, as well as the number of exceptions. The worst loss recorded by the Risk Metrics method is that of the Russian (-4.181%) at the 99% threshold. Among the 2000 observations, 32 performance values (losses) exceed the VaR.

4.1.4 GARCH

We describe in the following the different steps of the application of the GARCH method to estimate the VaR.

Step 1: download the data.

We download the adjusted closing prices of market indices from January 1, 2011, to December 31, 2018, using Yahoo Finance. Since we have missing data, we use the na omit command. This function removes all incomplete cases from the data (see ***Annex 5***).

Step 2: Obtain data returns

Based on the daily returns, we can conclude that high volatility days are followed by high volatility days and low volatility days by low volatility days (See ***Annex 6***).

Step 3: Find the best model using ARIMA

In this step, it is a question of finding the best ARIMA model (p, d, q) based on the Bayesian information criteria. Note that the returns of a financial series are always stationary and therefore integrated of order 0 I (0). Therefore, ARIMA is in fact only an ARMA (p, q) process (See ***Annex 7***).

Step 4: Testing the ARCH effect

To validate the ARIMA-type modelling, it is necessary to test the absence of heteroskedasticity through the ARCH effect. To do this, it is a question of applying the Ljung-Box test on the first 12 shifts of the squared residuals of the best ARIMA model under the null hypothesis of no ARCH effect. (See ***Anne***x ***7***)

If the value of p of the Ljung-Box test is less than 5% of significance, the ARCH effect is indeed present and the modulization of the GARCH type is then essential (See ***Annex 7***).

Step 5: Development of a GARCH model

For the sake of simplicity, we apply a GARCH (0,1) type modelling. For the GARCH theory, we specify the object called res_garch01_spec, in which we want to develop a GARCH (p, q) on ARIMA (p, 0, q).

Step 6: Backtesting the risk model

Once the GARCH model has been estimated, we verify the performance of the model by performing a historical backtest. To do this, we can compare the estimated VaR (value at risk) with the actual return over the period. If the return is more negative than the VaR, we have exceeded the VaR. In our case, exceeding the VaR should only occur 1% of the time if we have specified a confidence level of 99%, and 5% of the cases if we have specified a confidence level of 95%. The 1% and 5% VaR show the 1% and 5% probability of its extreme loss. (See **Annex *8***)

Table 4 shows the number of exceptions and the total number of observations, which differ between countries due to missing data.

The GARCH method records very high numbers of exceptions for the 5 countries, a sign of an inaccuracy in the estimate of the VaR by this method.

Table 4: Estimation VaR by the GARCH model

Pays	Confidence level	Number of exceptions	Number of total observations
Brazil	95% (α=5%)	115	1854
	99% (α=1%)	32	1854
Russia	95% (α=5%)	84	1310
	99% (α=1%)	20	1310
India	95% (α=5%)	149	1840
	99% (α=1%)	72	1840
China	95% (α=5%)	101	1824
	99% (α=1%)	49	1824
South Africa	95% (α=5%)	140	1965
	99% (α=1%)	50	1965

Source: Survey Data, 2022

4.2 Backtesting Value at Risk Forecast: Kupiec Pof-Test

Table 5: Percentages of exceptions: A comparative result

	95%				99%			
	HS	**Historical Method**	**Risk metrics**	**GARCH**	**HS**	**Historical Method**	**Risk metrics**	**GARCH**
Brazil	4.94%	4,957%	4.65%	6,2%	0.91%	0,961%	1.01%	3.6%
Russia	5.32%	4,950%	4.15%	6.4%	1.53%	0,950%	1.6%	1.5%
India	4.51%	4,970%	4.96%	8.1%	1.10%	0,963%	1.52%	3.9%
China	4.56%	4,936%	4.21%	5.5%	1.39%	0.925%	2.05%	2.7%
South Africa	5.85%	4,938%	4.94%	7.1%	0.96%	0,959%	1.72%	2.5%

Source: Survey Data, 2022

The percentages indicated in the table reflect the percentages of rejections of the null hypothesis. For the more liberal level of coverage (α = 5%), GARCH has the worst performance for most countries, while HS fails in Russia and South Africa. In addition, Risk Metrics dominate the three other methods. When a more conservative level of coverage is considered (α = 1%), the historical method has shown the best overall performance, more clearly outperforming GARCH and Risk Metrics. In general, we get a higher percentage of VaR method for more liberal coverage levels.

Once the one-day VaR and the number of exceptions for each confidence level are known, the likelihood ratio test must be calculated.

If the calculated LR exceeds the critical value, the null hypothesis and the model accuracy must be rejected for a certain level of confidence.

The null hypothesis indicates that the observed failure percentage equals the failure rate, which is suggested by the confidence interval. Moreover, the purpose of accepting the null hypothesis is to prove that the model is accurate. In the case where the quantity of likelihood ratio is greater than the critical value of χ^2, the conclusion on the rejection of the null hypothesis and the inaccuracy of the model would be made.

The "LR" likelihood ratio test is expressed according to the following formula:

$$LR\ POF = -2\ ln \frac{((1-p)^{T-x} * p^{x})}{\left[1-(\frac{x}{T})\right]^{T-x} * \left(\frac{x}{T}\right)^{x}} \tag{7}$$

Where:

p: Confidence level

T: Total number of observations

x: Number of exceptions

According to Jorion (2001), "the likelihood ratio is a statistical test that calculates the ratio between the maximum probabilities of a result under two alternative hypotheses. The maximum probability of the result observed under the null hypothesis is defined in the numerator and the maximum probability of the result observed under the alternative hypothesis is defined in the denominator. The decision is then based on the value of this ratio. The smaller the ratio, the larger the LR statistic will be. If the value becomes too large and greater than the critical value of the chi-square distribution, the null hypothesis is rejected. According to statistical decision theory, the likelihood ratio test is the most powerful test in its class".

As we have already specified for the POF test, the calculation of the likelihood test is necessary. Thus, it can be calculated by plugging the appropriate data from the table (1,2,3 and 4) into the likelihood ratio formula. This means that strong evidence is needed to reject the null hypothesis and the accuracy of the model. To draw a valid conclusion about the validity of the model, the critical value at the two levels 5% and 1% are determined from the chi-square table; the two values are 3.84 at level 5% and 6.63 at level 1%. (***Annex 9)***

Table 6: Kupiec-POF test results

	95%				99%			
	HS	**Risk metrics**	**Historical Method**	**GARCH**	**HS**	**Risk metrics**	**Historical Method**	**GARCH**
Brazil	Accepted	Accepted	Accepted	Reject	Accepted	Accepted	Accepted	Reject
Russia	Accepted	Accepted	Accepted	Reject	Accepted	Accepted	Accepted	Accepted
India	Accepted	Accepted	Accepted	Reject	Accepted	Accepted	Accepted	Reject
China	Accepted	Accepted	Accepted	Accepted	Accepted	Reject	Accepted	Reject
South Africa	Accepted	Accepted	Accepted	Reject	Accepted	Reject	Accepted	Reject

Source: Survey Data, 2022

The test used for the Backtesting of the amount of VaR expected in this research is a so-called failure proportion test. This test only considers the number of exceptions, not when the exception occurs. Therefore, the number of exceptions is critical information necessary for the model to be accurate or not (whether the null hypothesis is rejected or accepted).

- If we refer to the historical simulation method and to the historical method, the two methods are reliable for all countries at levels of 95% and 99% thresholds, while the difference lies in the Risk metrics method, which underestimates the risk at levels 99% level for China and South Africa. In comparison, the GARCH method gives a poor estimate for both thresholds and for all countries, except at level 95% for China and at 99% for Russia.
- If we seek to estimate the risk in Brazil and India, we must apply either the historical method or historical simulation or Risk metrices. These methods gave us a satisfactory estimate at levels 99% and 95%.
- The only method that should not be applied for risk measurement in Russia is GARCH.
- The risk estimates for China at level 99% can be applied by the four methods of measuring VaR. While at level 95%, the GARCH method can no longer be used because, according to the POF test, this method is no longer reliable.

- The results obtained from these four methods and after the application of the validation test, the VaR at level 95% must be estimated by the historical method, Risk metrics and by historical simulation, while at level 99%, the estimation is made by historical simulation and by the historical method.

5. Conclusion

The variety of risk measurement approaches that have been developed in the financial market over the last decades raises a question about the validity of these measurements. One of the most popular measures in the literature is the value at risk (VaR). Knowing the accuracy of the measurement is especially important for financial institutions, as they use VaR to estimate the amount of liquidity they need to reserve to cover potential losses. Any disability in the VaR model can mean that the institution does not hold sufficient reserves and could lead to significant losses, not only for the institution but potentially for its depositors and retail investors.

This research implements a VaR analysis for the BRICS countries (Brazil, Russia, India, China, and Africa from South) stock markets with market indices that represent the most relevant stocks in these countries. In addition, different performance measures for the assessment of the estimated VaR were discussed. The objective was to study the reliability of four methods (Historical Simulation, Risk metrics, Historical Method, GARCH) in estimating market VaR.

The use of backtesting is a primary task, which consists in comparing the measure of the calculated VaR with the real losses (or gains) realized by the portfolio by the index. A Backtest is based on the level of confidence assumed in the calculation.

The results showed that in the five countries and at different levels of trust, the Historical Method and Historical Simulation were the most robust. The change of country and threshold has no effect on the reliability of the VaR estimate. This means that there were two methods to estimate risk in emerging BRICS markets. While the GARCH model arrived last, it failed for all countries.

The results were obtained following the Kupiec POF Backtesting, but they can be confirmed by other tests, such as the Kupiec TUFF test (1995), the test of independence of Christoffersen (1998), DBI of Christofferssen and Pelletier (2004) and the DQ Engle and Manganelli test (2004).

As a future line of research, it would be interesting to apply these methods to the ES (Expected Shortfall), which has become increasingly important in the field of financial market risk measurement. It is an alternative to value at risk, which is more sensitive to the shape of the tail of the loss distribution. In addition, it would be useful to extend our analysis with additional VaR forecasting methods such as Monte Carlo simulation, Parametric Method, and EVT (Extreme Value Theory).

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Annexures:

Annex 1: Market portfolios sorted by capitalization

	A	B	C	D	E
1	RUSSIE 20/50	INDE 20/30	CHINE 20/1474	AFRIQUE DE SUD 20/40	BRAZIL 20/50
2	INTER RAO YEES	SUN PHARM.INDUSTRIES	BANK OF COMMS.'A'	NASPERS	VALE ON
3	GAZPROM	TATA CONSULTANCY SVS	BANK OF CHINA 'A'	ASPEN PHMCR.HDG.	ITAU UNIBANCO HOLDING PN
4	MMC NORILSK NICKEL	TATA MOTORS	ALUMINUM CORP.OF CHINA 'A'	LONMIN (JSE)	AMBEV ON
5	AEROFLOT RUSS.AIRL.	TATA STEEL	AIR CHINA LIMITED 'A'	EXXARO RESOURCES	PETROLEO BRASILEIRO ON
6	NK LUKOIL	RELIANCE INDUSTRIES	AN INSURANCE (GP.) CO. OF CHIN	SASOL	BANCO BRADESCO PN
7	FSK YEES	MARUTI SUZUKI INDIA	CITIC SECURITIES 'A'	ANGLOGOLD ASHANTI	BANCO DO BRASIL ON
8	SEVERSTAL	VEDANTA	BAOSHAN IRON & STL.'A'	BIDVEST GROUP	RAIA DROGASIL ON
9	TATNEFT	HDFC BANK	KWEICHOW MOUTAI 'A'	MONDI	ESTACIO PARTICIPACOES ON
10	NOVATEK	HOUSING DEVELOPMENT FINANCE CORPORATION	JIANGXI CPR. 'A'	MTN GROUP	TELEFONICA BRASIL PN
11	URALKALI	BAJAJ FINANCE	EVELOPMENTS AND HOLDINGS GR	HARMONY GOLD MNG.	B3 BRASIL BOLSA BALCAO ON
12	POLYUS	LARSEN & TOUBRO	YOUNGOR GROUP 'A'	REINET INVESTMENTS (JSE) SCA	EQUATORIAL ENERGIA ON
13	NOVOLIPETSK STEEL	HERO MOTOCORP	BEIJING GEHUA CATV NET. 'A'	VODACOM GROUP	BRF BRASIL FOODS ON
14	MOBILE TELESYSTEMS	AXIS BANK	INNER MONGOLIA YILI INDL.GP.'A	FIRSTRAND	GOL LINHAS AEREAS INTELIGENTES PN
15	MECHEL OAO	ASIAN PAINTS	SAIC MOTOR 'A'	KUMBA IRON ORE	QUALICORP ON
16	SURGUTNEFTEGAS	MAHINDRA & MAHINDRA	DAQIN RAILWAY 'A'	SHOPRITE	BB SEGURIDADE ON
17	ROSTELECOM	HCL TECHNOLOGIES	SHAI.PUDONG DEV.BK. 'A'	NEDBANK GROUP	GERDAU PN
18	SBERBANK OF RUSSIA	ITC	IDUSTRIAL & COML.BK.OF CHINA '	ANGLO AMERICAN PLATINUM	ITAUSA INVESTIMENTOS ITAU PN
19	PIK GROUP	KOTAK MAHINDRA BANK	OFFS.OIL ENGR. 'A'	IMPALA PLATINUM	LOJAS RENNER ON
20	ROSSETI	INFOSYS	HUANENG POWER INTL.'A'	STANDARD BANK GROUP	LOJAS AMERICANAS PN REP1 PN
21	OC ROSNEFT	HINDUSTAN UNILEVER	CHINA YANGTZE PWR. 'A'	STEINHOFF INTL.HOLDING	RUMO ON

Annex 2: Calculation of VaR by historical simulation

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	
3															
4															
5	N°	DATE	NASPERS	ASPEN PHMCR.HDG.	LONMIN (JSE)	EXXARO RESOURCES	SASOL	ANGLOGOLD ASHANTI	BIDVEST GROUP	MONDI	MTN GROUP	HARMONY GOLD MNG.	REINET INVESTMENTS (JSE) SCA	VODACOM GROUP	FIRS
6	1	31/12/2018	2811,84	134,82	8,39	137,87	425	181,75	206,85	309,57	89	25,2	218,45	132	
7	2	28/12/2018	2811,84	133,31	8,06	133,46	426,74	180,84	204	306,5	88,06	24,94	217,78	131,6	
8	3	27/12/2018	2779,55	130	8,19	132,15	419,85	182,7	197,63	302,44	87,79	25,03	207,47	128,42	
9	4	26/12/2018	2866,43	129,5	7,9	134,07	422	184	203,16	309	85,5	24,8	211	127,78	
10	5	25/12/2018	2866,43	129,5	7,9	134,07	422	184	203,16	309	85,5	24,8	211	127,78	
11	6	24/12/2018	2866,43	129,5	7,9	134,07	422	184	203,16	309	85,5	24,8	211	127,78	
12	7	21/12/2018	2792,45	129,72	7,9	132,5	411,32	176,22	201,85	301,87	83,68	24,22	204,08	129,08	
13	8	20/12/2018	2717,27	132,29	7,92	134,65	412,4	179,9	205,37	301,98	84,78	25,06	206,7	127,52	
14	9	19/12/2018	2734,27	133,17	8,73	134,31	417,16	179,47	203,91	298,79	85,7	25,9	207,48	130,62	
15	10	18/12/2018	2737,69	134,58	8,97	139,2	407,91	175,99	198,5	296,24	84,13	23,96	213,6	123,9	
16	11	17/12/2018	2732,59	129,5	8,96	138,58	428,63	170,82	205,22	300,09	86,01	22,81	216,44	124,3	
17	12	14/12/2018	2732,59	129,5	8,96	138,58	428,63	170,82	205,22	300,09	86,01	22,81	216,44	124,3	
18	13	13/12/2018	2734,28	141,64	8,59	138,46	426,94	170,78	204,5	300,14	84,95	23,27	217,1	124,49	
19	14	12/12/2018	2770,29	142,76	8,52	129,3	424,07	167,36	198,83	309,5	85,94	23,23	220,95	123,7	
20	15	11/12/2018	2764,44	142,04	8,62	127,63	423,56	164,5	193,02	305,26	83,62	22,85	226,14	123,51	
21	16	10/12/2018	2666,4	138,32	8,59	125,88	413,69	161,54	193,22	301,79	83,89	23,62	215,6	120	
22	17	07/12/2018	2728,45	144,55	8,6	128,19	431,44	153,42	199,89	299,06	86,39	23,11	212,21	123,16	
23	18	06/12/2018	2671,25	146,73	8,45	129,46	425,5	153,95	201,5	306,05	86	22,99	213,13	122,66	
24	19	05/12/2018	2785,29	149,9	8,26	132	431,5	147,46	209,95	310,5	85,85	22	207,23	125,17	
25	20	04/12/2018	2840,93	148,4	8,09	131,97	431,36	144,4	206,5	310	86,74	21,19	205,03	125,85	
26	21	03/12/2018	2800,7	146,5	7,95	132	434	139,66	202,57	314	87,22	21	203,58	125,78	
27	22	30/11/2018	2679	147,08	7,86	129,19	406,28	139,13	205,3	305	87,38	20,87	202,19	125,5	
28	23	29/11/2018	2734,27	151,11	8,1	132,5	421,07	140,51	209,35	312,01	89,8	21,6	210	128,25	
29	24	28/11/2018	2806,54	149,77	8,01	132,08	420,24	139,45	203,3	313,72	88,4	21,26	213,82	123,9	
30	25	27/11/2018	2705,18	150,07	8,13	135	412,67	139,53	199,31	313,21	88,14	21,75	210,34	125,1	
31	26	26/11/2018	2736,39	154,01	8,07	132,5	415,62	141,6	200,29	316,55	89,77	22,38	206	124	

3																	
4	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
5	NASPERS	EN PHMCR.H	ONMIN (JSE	ARO RESOUR	SASOL	LOGOLD ASH	DVEST GROU	MONDI	MTN GROUP	MONY GOLD	IVESTMENTS	DACOM GRC	FIRSTRAND	MBA IRON C	SHOPRITE	NEDBANK GROUP	NGLO AMERICAN PLATINU	IMPAL
6	0,00%	1,13%	4,01%	3,25%	-0,41%	0,50%	1,39%	1,00%	1,06%	1,04%	0,31%	0,30%	0,44%	1,08%	0,29%	0,26%	-1,04%
7	1,16%	2,51%	-1,60%	0,99%	1,63%	-1,02%	3,17%	1,33%	0,31%	-0,36%	4,85%	2,45%	1,64%	3,44%	0,33%	1,97%	0,29%
8	-3,08%	0,39%	3,61%	-1,44%	-0,51%	-0,71%	-2,76%	-2,15%	2,64%	0,92%	-1,69%	0,50%	0,94%	-1,09%	1,18%	0,89%	-0,50%
9	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
10	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
11	2,61%	-0,17%	0,00%	1,18%	2,56%	4,32%	0,65%	2,33%	2,15%	2,37%	3,33%	-1,01%	0,73%	1,80%	2,06%	-0,36%	1,06%
12	2,73%	-1,96%	-0,25%	-1,61%	-0,26%	-2,07%	-1,73%	-0,04%	-1,31%	-3,41%	-1,28%	1,22%	-1,40%	-1,27%	1,27%	-0,42%	-0,19%
13	-0,62%	-0,66%	-9,74%	0,25%	-1,15%	0,24%	0,71%	1,06%	-1,08%	-3,30%	-0,38%	-2,40%	0,45%	0,37%	-3,15%	0,35%	-0,19%
14	-0,13%	-1,05%	-2,71%	-3,58%	2,24%	1,96%	2,69%	0,86%	1,85%	7,79%	-2,91%	5,28%	1,28%	-1,57%	0,23%	2,98%	-1,01%
15	0,19%	3,85%	0,11%	0,45%	-4,95%	2,98%	-3,33%	-1,29%	-2,21%	4,92%	-1,32%	-0,32%	-2,45%	2,40%	-2,26%	-2,21%	3,07%
16	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
17	-0,06%	-8,96%	4,22%	0,09%	0,40%	0,02%	0,35%	-0,02%	1,24%	-2,00%	-0,30%	-0,15%	-1,84%	-1,23%	-0,75%	0,45%	-0,56%
18	-1,31%	-0,79%	0,82%	6,84%	0,67%	2,02%	2,81%	-3,07%	-1,16%	0,17%	-1,76%	0,64%	-0,17%	2,38%	2,20%	1,20%	2,50%
19	0,21%	0,51%	-1,17%	1,30%	0,12%	1,72%	2,97%	1,38%	2,74%	1,65%	-2,32%	0,15%	1,86%	-0,49%	3,38%	0,40%	3,94%
20	3,61%	2,65%	0,35%	1,38%	2,36%	1,82%	-0,10%	1,14%	-0,32%	-3,31%	4,77%	2,88%	0,93%	-0,03%	-0,93%	1,28%	0,66%
21	-2,30%	-4,41%	-0,12%	-1,82%	-4,20%	5,16%	-3,39%	0,91%	-2,94%	2,18%	1,58%	-2,60%	-2,16%	-2,12%	-3,26%	-2,24%	2,68%
22	2,12%	-1,50%	1,76%	-0,99%	1,39%	-0,34%	-0,80%	-2,31%	0,45%	0,52%	-0,43%	0,41%	0,23%	4,67%	-1,02%	1,78%	-0,34%
23	-4,18%	-2,14%	2,27%	-1,94%	-1,40%	4,31%	-4,11%	-1,44%	0,17%	4,40%	2,81%	-2,03%	-3,05%	-0,20%	-2,07%	-3,19%	3,91%
24	-1,98%	1,01%	2,08%	0,02%	0,03%	2,10%	1,66%	0,16%	-1,03%	3,75%	1,07%	-0,54%	-1,62%	3,48%	-1,47%	-2,71%	-0,08%
25	1,43%	1,29%	1,75%	-0,02%	-0,61%	3,34%	1,92%	-1,28%	-0,55%	0,90%	0,71%	0,06%	-0,50%	-1,11%	0,56%	1,21%	-0,42%
26	4,44%	-0,40%	1,14%	2,15%	6,60%	0,38%	-1,34%	2,91%	-0,18%	0,62%	0,69%	0,22%	2,82%	2,88%	-0,20%	1,65%	5,20%
27	-2,04%	-2,70%	-3,01%	-2,53%	-3,58%	-0,99%	-1,95%	-2,27%	-2,73%	-3,44%	-3,79%	-2,17%	-4,78%	-2,96%	-1,68%	-2,59%	-2,02%
28	-2,61%	0,89%	1,12%	0,32%	0,20%	0,76%	2,93%	-0,55%	1,57%	1,59%	-1,80%	3,45%	1,26%	-1,94%	0,04%	1,73%	1,27%
29	3,68%	-0,20%	-1,49%	-2,19%	1,82%	-0,06%	1,98%	0,16%	0,29%	-2,28%	1,64%	-0,96%	0,13%	-0,93%	-0,79%	1,56%	-0,28%

	AY	AZ	BA	BB	BC	BD	BE	BF	BG	BH
1										
2										
3										
4										
5	DVEST GROU	MONDI	MTN GROUP	MONY GOLD	IVESTMENTS	DACOM GRC	FIRSTRAND	MBA IRON C	SHOPRITE	DBANK GRO
6	1,39 ZAR	1,00 ZAR	1,06 ZAR	1,04 ZAR	0,31 ZAR	0,30 ZAR	0,44 ZAR	1,08 ZAR	0,29 ZAR	0,26 ZAR
7	3,17 ZAR	1,33 ZAR	0,31 ZAR	- 0,36 ZAR	4,85 ZAR	2,45 ZAR	1,64 ZAR	3,44 ZAR	0,33 ZAR	1,97 ZAR
8	- 2,76 ZAR	- 2,15 ZAR	2,64 ZAR	0,92 ZAR	- 1,69 ZAR	0,50 ZAR	0,94 ZAR	- 1,09 ZAR	1,18 ZAR	0,89 ZAR
9	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR
10	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR
11	0,65 ZAR	2,33 ZAR	2,15 ZAR	2,37 ZAR	3,33 ZAR	- 1,01 ZAR	0,73 ZAR	1,80 ZAR	2,06 ZAR	- 0,36 ZAR
12	- 1,73 ZAR	- 0,04 ZAR	- 1,31 ZAR	- 3,41 ZAR	- 1,28 ZAR	1,22 ZAR	- 1,40 ZAR	- 1,27 ZAR	1,27 ZAR	- 0,42 ZAR
13	0,71 ZAR	1,06 ZAR	- 1,08 ZAR	- 3,30 ZAR	- 0,38 ZAR	- 2,40 ZAR	0,45 ZAR	0,37 ZAR	- 3,15 ZAR	0,35 ZAR
14	2,69 ZAR	0,86 ZAR	1,85 ZAR	7,79 ZAR	- 2,91 ZAR	5,28 ZAR	1,28 ZAR	- 1,57 ZAR	0,23 ZAR	2,98 ZAR
15	- 3,33 ZAR	- 1,29 ZAR	- 2,21 ZAR	4,92 ZAR	- 1,32 ZAR	- 0,32 ZAR	- 2,45 ZAR	2,40 ZAR	- 2,26 ZAR	- 2,21 ZAR
16	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR	- ZAR
17	0,35 ZAR	- 0,02 ZAR	1,24 ZAR	- 2,00 ZAR	- 0,30 ZAR	- 0,15 ZAR	- 1,84 ZAR	- 1,23 ZAR	- 0,75 ZAR	0,45 ZAR
18	2,81 ZAR	- 3,07 ZAR	- 1,16 ZAR	0,17 ZAR	- 1,76 ZAR	0,64 ZAR	- 0,17 ZAR	2,38 ZAR	2,20 ZAR	1,20 ZAR
19	2,97 ZAR	1,38 ZAR	2,74 ZAR	1,65 ZAR	- 2,32 ZAR	0,15 ZAR	1,86 ZAR	- 0,49 ZAR	3,38 ZAR	0,40 ZAR
20	- 0,10 ZAR	1,14 ZAR	- 0,32 ZAR	- 3,31 ZAR	4,77 ZAR	2,88 ZAR	0,93 ZAR	- 0,03 ZAR	- 0,93 ZAR	1,28 ZAR
21	- 3,39 ZAR	0,91 ZAR	- 2,94 ZAR	2,18 ZAR	1,58 ZAR	- 2,60 ZAR	- 2,16 ZAR	- 2,12 ZAR	- 3,26 ZAR	- 2,24 ZAR
22	- 0,80 ZAR	- 2,31 ZAR	0,45 ZAR	0,52 ZAR	- 0,43 ZAR	0,41 ZAR	0,23 ZAR	4,67 ZAR	- 1,02 ZAR	1,78 ZAR
23	- 4,11 ZAR	- 1,44 ZAR	0,17 ZAR	4,40 ZAR	2,81 ZAR	- 2,03 ZAR	- 3,05 ZAR	- 0,20 ZAR	- 2,07 ZAR	- 3,19 ZAR
24	1,66 ZAR	0,16 ZAR	- 1,03 ZAR	3,75 ZAR	1,07 ZAR	- 0,54 ZAR	- 1,62 ZAR	3,48 ZAR	- 1,47 ZAR	- 2,71 ZAR
25	1,92 ZAR	- 1,28 ZAR	- 0,55 ZAR	0,90 ZAR	0,71 ZAR	0,06 ZAR	- 0,50 ZAR	- 1,11 ZAR	0,56 ZAR	1,21 ZAR
26	- 1,34 ZAR	2,91 ZAR	- 0,18 ZAR	0,62 ZAR	0,69 ZAR	0,22 ZAR	2,82 ZAR	2,88 ZAR	- 0,20 ZAR	1,65 ZAR
27	- 1,95 ZAR	- 2,27 ZAR	- 2,73 ZAR	- 3,44 ZAR	- 3,79 ZAR	- 2,17 ZAR	- 4,78 ZAR	- 2,96 ZAR	- 1,68 ZAR	- 2,59 ZAR
28	2,93 ZAR	- 0,55 ZAR	1,57 ZAR	1,59 ZAR	- 1,80 ZAR	3,45 ZAR	1,26 ZAR	- 1,94 ZAR	0,04 ZAR	1,73 ZAR
29	1,98 ZAR	0,16 ZAR	0,29 ZAR	- 2,28 ZAR	1,64 ZAR	- 0,96 ZAR	0,13 ZAR	- 0,93 ZAR	- 0,79 ZAR	1,56 ZAR

	BI	BJ	BK	BL	BM	BN	BO	BP	BQ
1									
2								95%	99%
3					La Value at risk à 99%=	61,0073692	Exception=	122	20
4					La Value at risk à 95%=	35,384095	Exception(%)=	5,85%	0,96%
5	MERICAN PL	PALA PLATIN	ARD BANK (	OFF INTL.HOLDING		portfolio	test de Var (95%)	test de VaR(99%)	
6	- 1,04 ZAR	- 0,05 ZAR	0,32 ZAR	1,17 ZAR		16,05 ZAR	0	0	
7	0,29 ZAR	1,93 ZAR	1,69 ZAR	3,59 ZAR		30,28 ZAR	0	0	
8	- 0,50 ZAR	6,01 ZAR	- 0,38 ZAR	- 4,18 ZAR		- 1,40 ZAR	0	0	
9	- ZAR	- ZAR	- ZAR	- ZAR		- ZAR	0	0	
10	- ZAR	- ZAR	- ZAR	- ZAR		- ZAR	0	0	
11	1,06 ZAR	- 0,38 ZAR	0,59 ZAR	4,18 ZAR		29,99 ZAR	0	0	
12	- 0,19 ZAR	- 4,23 ZAR	- 0,94 ZAR	0,61 ZAR		- 16,53 ZAR	0	0	
13	- 0,19 ZAR	- 3,49 ZAR	- 0,60 ZAR	- 1,82 ZAR		- 25,14 ZAR	0	0	
14	- 1,01 ZAR	- 0,30 ZAR	3,90 ZAR	- 2,38 ZAR		15,42 ZAR	0	0	
15	3,07 ZAR	2,72 ZAR	- 1,63 ZAR	- 2,90 ZAR		- 4,20 ZAR	0	0	
16	- ZAR	- ZAR	- ZAR	- ZAR		- ZAR	0	0	
17	- 0,56 ZAR	- 4,71 ZAR	- 0,39 ZAR	1,15 ZAR		- 13,07 ZAR	0	0	
18	2,50 ZAR	2,18 ZAR	1,51 ZAR	- 2,85 ZAR		14,85 ZAR	0	0	
19	3,94 ZAR	1,56 ZAR	2,70 ZAR	15,13 ZAR		37,73 ZAR	0	0	
20	0,66 ZAR	- 2,86 ZAR	- 0,35 ZAR	2,65 ZAR		18,59 ZAR	0	0	
21	2,68 ZAR	6,31 ZAR	- 2,30 ZAR	- 4,59 ZAR		- 19,62 ZAR	0	0	
22	- 0,34 ZAR	3,61 ZAR	- 0,70 ZAR	- 2,53 ZAR		5,97 ZAR	0	0	
23	3,91 ZAR	4,42 ZAR	- 3,35 ZAR	-10,66 ZAR		- 17,46 ZAR	0	0	
24	- 0,08 ZAR	- 3,65 ZAR	- 1,21 ZAR	- 1,12 ZAR		- 0,05 ZAR	0	0	
25	- 0,42 ZAR	- 1,07 ZAR	0,70 ZAR	- ZAR		8,30 ZAR	0	0	
26	5,20 ZAR	2,57 ZAR	2,48 ZAR	1,12 ZAR		35,76 ZAR	0	0	
27	- 2,02 ZAR	- 1,03 ZAR	- 2,79 ZAR	- 2,77 ZAR		- 51,81 ZAR	1	0	
28	1,27 ZAR	1,12 ZAR	1,51 ZAR	- 2,16 ZAR		10,70 ZAR	0	0	
29	- 0,28 ZAR	1,13 ZAR	2,40 ZAR	- 1,06 ZAR		4,55 ZAR	0	0	

Annex 3: Calculation of VaR by the historical method

	A	B	C	D	E	F	G	H	I
1			FTSE/JSE ALL SHARE_trade close						
2	N°	code	JSE(TC)	gains	gains triés	Ref VaR	Var à x% =	Exceptions95%	Exceptions 99%
3	1	12/31/18	46726,5886	232,799804	-1936,68771	0,047938639	99,95	0	0
4	2	12/28/18	46493,78879	838,342912	-1883,13595	0,095877277	99,90	0	0
5	3	12/27/18	45655,44588	-548,422317	-1706,50561	0,143815916	99,86	0	0
6	4	12/26/18	46203,8682	0	-1642,23226	0,191754554	99,81	0	0
7	5	12/25/18	46203,8682	0	-1427,64359	0,239693193	99,76	0	0
8	6	12/24/18	46203,8682	649,472737	-1411,87584	0,287631831	99,71	0	0
9	7	12/21/18	45554,39546	201,537747	-1390,74653	0,33557047	99,66	0	0
10	8	12/20/18	45352,85771	-184,718824	-1380,39371	0,383509108	99,62	0	0
11	9	12/19/18	45537,57654	310,421364	-1370,33453	0,431447747	99,57	0	0
12	10	12/18/18	45227,15517	-301,710863	-1355,42437	0,479386385	99,52	0	0
13	11	12/17/18	45528,86604	0	-1345,25304	0,527325024	99,47	0	0
14	12	12/14/18	45528,86604	-110,134501	-1345,07031	0,575263663	99,42	0	0
15	13	12/13/18	45639,00054	35,8152521	-1337,54621	0,623202301	99,38	0	0
16	14	12/12/18	45603,18528	368,855126	-1291,80539	0,67114094	99,33	0	0
17	15	12/11/18	45234,33016	816,629891	-1277,98213	0,719079578	99,28	0	0
18	16	12/10/18	44417,70027	-587,607409	-1261,61082	0,767018217	99,23	0	0
19	17	12/07/18	45005,30768	226,554853	-1255,31643	0,814956855	99,19	0	0
20	18	12/06/18	44778,75282	-896,160813	-1229,37	0,862895494	99,14	1	0
21	19	12/05/18	45674,91364	-489,937258	-1192,32	0,910834132	99,09	0	0
22	20	12/04/18	46164,85089	112,495684	-1163,76432	0,958772771	99,04	0	0
23	21	12/03/18	46052,35521	1395,46634	-1161,52427	1,006711409	98,99	0	0
24	22	11/30/18	44656,88887	-1046,28606	-1145,70329	1,054650048	98,95	1	0
25	23	11/29/18	45703,17494	-95,9005523	-1097,96176	1,102588686	98,90	0	0
26	24	11/28/18	45799,07549	574,43139	-1061,05496	1,150527325	98,85	0	0
27	25	11/27/18	45224,6441	-320,53686	-1046,28606	1,198465964	98,80	0	0

K	L	M
	95%	99%
VaR=	-680,9632462	-1161,52427
Exceptions=	103	20
Exceptons %	4,938%	0,959%

Annex 4: Calculation of VaR by risk metrics:

- Brazil

	A	B	C	D	E	F	G	H	I	J
1	N°	date	BOVESPA(TC)	returns	résiduals					
2	1	01/03/11	69962,32			returns square	variance	standard deviation	95%	99%
3	2	01/04/11	70317,79	0,51%		0,0026%	0,021%	1,452%	-2,388%	-3,378%
4	3	01/05/11	71091,03	1,09%		0,0120%				
5	4	01/06/11	70578,83	-0,72%		0,0052%				
6	5	01/07/11	70057,2	-0,74%		0,0055%				
7	6	01/10/11	70127,04	0,10%		0,0001%				
8	7	01/11/11	70423,44	0,42%		0,0018%				
9	8	01/12/11	71632,9	1,70%		0,0290%				
10	9	01/13/11	70721,44	-1,28%		0,0164%				
11	10	01/14/11	70940,22	0,31%		0,0010%				
12	11	01/17/11	70609,07	-0,47%		0,0022%				
13	12	01/18/11	70919,75	0,44%		0,0019%				
14	13	01/19/11	70058,08	-1,22%		0,0149%				
15	14	01/20/11	69561,53	-0,71%		0,0051%				
16	15	01/21/11	69133,09	-0,62%		0,0038%				
17	16	01/24/11	69426,57	0,42%		0,0018%				
18	17	01/26/11	68709,22	-1,04%		0,0108%				
19	18	01/27/11	68050,71	-0,96%		0,0093%				
20	19	01/28/11	66697,57	-2,01%		0,0403%				
21	20	01/31/11	66574,88	-0,18%		0,0003%				
22	21	02/01/11	67847,34	1,89%		0,0358%				
23	22	02/02/11	66688,48	-1,72%		0,0297%				
24	23	02/03/11	66764,84	0,11%		0,0001%				
25	24	02/04/11	65269,15	-2,27%		0,0513%				
26	25	02/07/11	65362,04	0,14%		0,0002%				
27	26	02/08/11	65771,33	0,62%		0,0039%				

- Russia

	A	B	C	D	E	F	G	H	I	J
1	N°	date	RTS (TC)	returns	residuals					
2	1	01/11/11	1802,23			returns squa	variance	standard deviation	95%	99%
3	2	01/12/11	1868,94	3,63%		0,1321%	0,032%	1,797%	-2,956%	-4,181%
4	3	01/13/11	1878,14	0,49%		0,0024%				
5	4	01/14/11	1870,09	-0,43%		0,0018%				
6	5	01/17/11	1901,61	1,67%		0,0279%				
7	6	01/18/11	1900,94	-0,04%		0,0000%				
8	7	01/19/11	1902,75	0,10%		0,0001%				
9	8	01/20/11	1868,46	-1,82%		0,0331%				
10	9	01/21/11	1884,76	0,87%		0,0075%				
11	10	01/24/11	1861,66	-1,23%		0,0152%				
12	11	01/25/11	1863,33	0,09%		0,0001%				
13	12	01/26/11	1894,2	1,64%		0,0270%				
14	13	01/27/11	1911,48	0,91%		0,0082%				
15	14	01/28/11	1885,53	-1,37%		0,0187%				
16	15	01/31/11	1870,31	-0,81%		0,0066%				
17	16	02/01/11	1910,01	2,10%		0,0441%				
18	17	02/02/11	1931,38	1,11%		0,0124%				
19	18	02/03/11	1917,07	-0,74%		0,0055%				
20	19	02/04/11	1928,58	0,60%		0,0036%				
21	20	02/07/11	1935,15	0,34%		0,0012%				
22	21	02/08/11	1910,5	-1,28%		0,0164%				
23	22	02/09/11	1900,28	-0,54%		0,0029%				
24	23	02/10/11	1846,92	-2,85%		0,0811%				
25	24	02/11/11	1881,9	1,88%		0,0352%				
26	25	02/14/11	1879,56	-0,12%		0,0002%				
27	26	02/15/11	1865,99	-0,72%		0,0053%				

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- India

	A	B	C	D	E	F	G	H	I	J
1	N°	Date	sensex(TC)	returns	Résiduals					
2	1	01/03/11	20561,05			returns square	variance	standard deviation	95%	99%
3	2	01/04/11	20498,72	-0,30%		0,0009%	0,009%	0,958%	-1,576%	-2,229%
4	3	01/05/11	20301,1	-0,97%		0,0094%				
5	4	01/06/11	20184,74	-0,57%		0,0033%				
6	5	01/07/11	19691,81	-2,47%		0,0611%				
7	6	01/10/11	19224,12	-2,40%		0,0578%				
8	7	01/11/11	19196,34	-0,14%		0,0002%				
9	8	01/12/11	19534,1	1,74%		0,0304%				
10	9	01/13/11	19182,82	-1,81%		0,0329%				
11	10	01/14/11	18860,44	-1,69%		0,0287%				
12	11	01/17/11	18882,25	0,12%		0,0001%				
13	12	01/18/11	19092,05	1,10%		0,0122%				
14	13	01/19/11	18978,32	-0,60%		0,0036%				
15	14	01/20/11	19046,54	0,36%		0,0013%				
16	15	01/21/11	19007,53	-0,21%		0,0004%				
17	16	01/24/11	19151,28	0,75%		0,0057%				
18	17	01/25/11	18969,45	-0,95%		0,0091%				
19	18	01/27/11	18684,43	-1,51%		0,0229%				
20	19	01/28/11	18395,97	-1,56%		0,0242%				
21	20	01/31/11	18327,76	-0,37%		0,0014%				
22	21	02/01/11	18022,22	-1,68%		0,0283%				
23	22	02/02/11	18090,62	0,38%		0,0014%				
24	23	02/03/11	18449,31	1,96%		0,0385%				
25	24	02/04/11	18008,15	-2,42%		0,0586%				
26	25	02/07/11	18037,19	0,16%		0,0003%				
27	26	02/08/11	17775,7	-1,46%		0,0213%				

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- China

	B	C	D	E	F	G	H	I	J
1	code	JSE(TC)	returns	Résiduals					
2	01/03/11	28830,78			returns square	variance	standard deviation	95%	99%
3	01/04/11	28933,58	0,36%		0,0013%	0,010%	1,021%	-1,680%	-2,376%
4	01/05/11	28502,97	-1,50%		0,0225%				
5	01/06/11	28608,82	0,37%		0,0014%				
6	01/07/11	28415,54	-0,68%		0,0046%				
7	01/10/11	28220,69	-0,69%		0,0047%				
8	01/11/11	28700,04	1,68%		0,0284%				
9	01/12/11	29038,72	1,17%		0,0138%				
10	01/13/11	29199,02	0,55%		0,0030%				
11	01/14/11	29226,96	0,10%		0,0001%				
12	01/17/11	28893,16	-1,15%		0,0132%				
13	01/18/11	29192,55	1,03%		0,0106%				
14	01/19/11	29013,19	-0,62%		0,0038%				
15	01/20/11	28514,94	-1,73%		0,0300%				
16	01/21/11	28808,74	1,03%		0,0105%				
17	01/24/11	28545,01	-0,92%		0,0085%				
18	01/25/11	28334,12	-0,74%		0,0055%				
19	01/26/11	28647,4	1,10%		0,0121%				
20	01/27/11	28812,39	0,57%		0,0033%				
21	01/28/11	28232,1	-2,03%		0,0414%				
22	01/31/11	28145,34	-0,31%		0,0009%				
23	02/01/11	28625,33	1,69%		0,0286%				
24	02/02/11	29274,22	2,24%		0,0502%				
25	02/03/11	29609,47	1,14%		0,0130%				
26	02/04/11	29718,96	0,37%		0,0014%				
27	02/07/11	29621,7	-0,33%		0,0011%				

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- South Africa

	A	B	C	D	E	F	G	H	I	J
1	N°	date	SHANG-SHG(TC)	Returns	Résiduals					
2	1	01/04/11	2852,648			returns square	variance	standard deviation	95%	99%
3	2	01/05/11	2838,593	-0,49%		0,00244%	0,019%	1,379%	-2,268%	-3,208%
4	3	01/06/11	2824,197	-0,51%		0,00259%				
5	4	01/07/11	2838,801	0,52%		0,00266%				
6	5	01/10/11	2791,809	-1,67%		0,02786%				
7	6	01/11/11	2804,047	0,44%		0,00191%				
8	7	01/12/11	2821,305	0,61%		0,00376%				
9	8	01/13/11	2827,713	0,23%		0,00051%				
10	9	01/14/11	2791,344	-1,29%		0,01676%				
11	10	01/17/11	2706,66	-3,08%		0,09491%				
12	11	01/18/11	2708,979	0,09%		0,00007%				
13	12	01/19/11	2758,097	1,80%		0,03229%				
14	13	01/20/11	2677,652	-2,96%		0,08762%				
15	14	01/21/11	2715,294	1,40%		0,01949%				
16	15	01/24/11	2695,72	-0,72%		0,00523%				
17	16	01/25/11	2677,432	-0,68%		0,00463%				
18	17	01/26/11	2708,814	1,17%		0,01358%				
19	18	01/27/11	2749,15	1,48%		0,02185%				
20	19	01/28/11	2752,75	0,13%		0,00017%				
21	20	01/31/11	2790,694	1,37%		0,01874%				
22	21	02/01/11	2798,96	0,30%		0,00087%				
23	22	02/09/11	2774,065	-0,89%		0,00798%				
24	23	02/10/11	2818,163	1,58%		0,02487%				
25	24	02/11/11	2827,328	0,32%		0,00105%				
26	25	02/14/11	2899,134	2,51%		0,06290%				
27	26	02/15/11	2899,237	0,00%		0,00000%				

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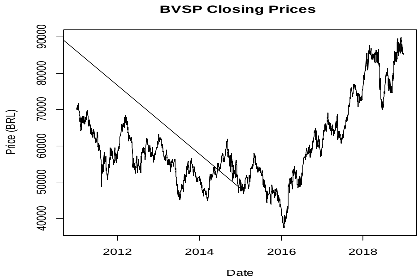
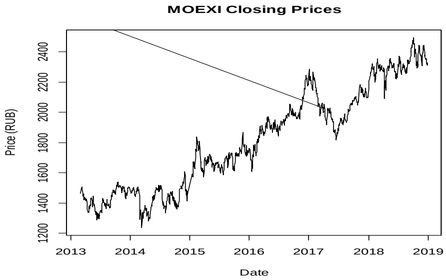
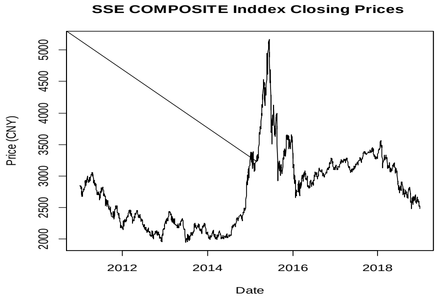
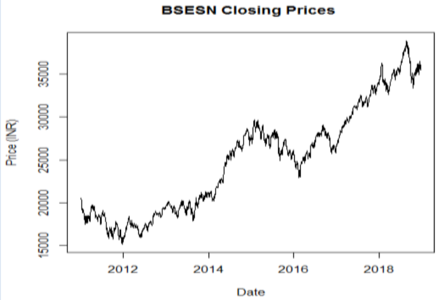
Annex 5: Volatility in the BRICS group's market indices

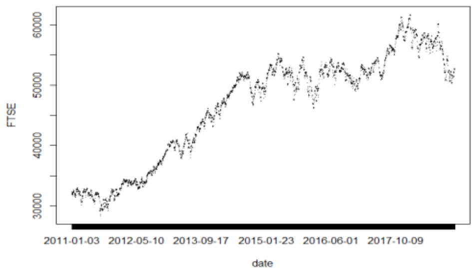




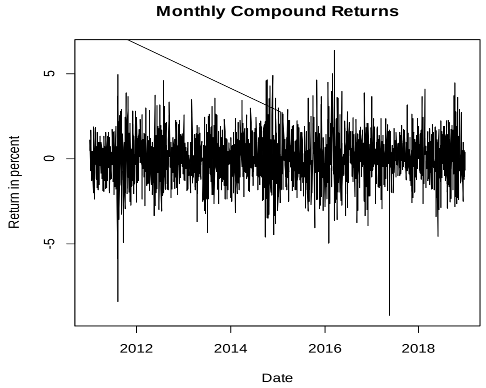


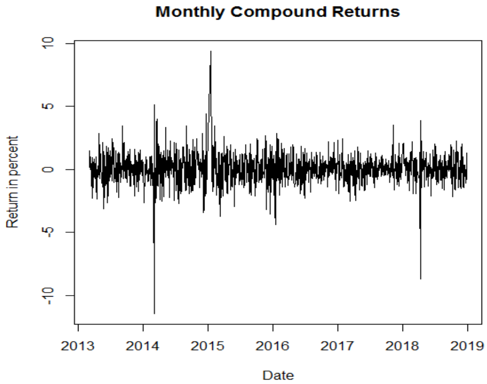


FTSE/JSE Closing Price



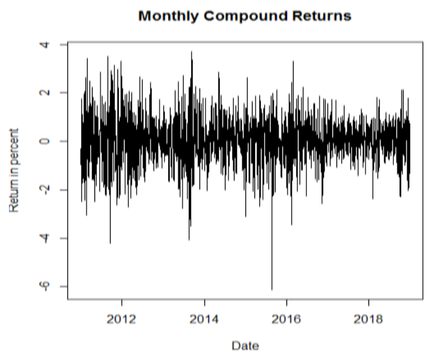
Annex 6: Daily returns of market indices

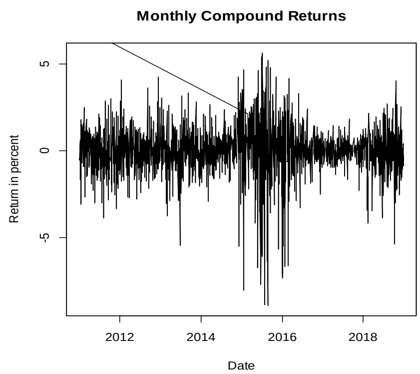




Brési

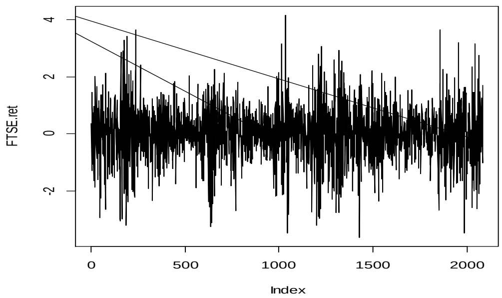
Russi





Inde

Chine



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Annex 7: The best average model using ARIMA and the ARCH effect test

- Brazil

```
> fit1 <- auto.arima(BVSPI.ret, trace=TRUE, test="kpss",  ic="bic")

 Fitting models using approximations to speed things up...

 ARIMA(2,0,2) with non-zero mean : Inf
 ARIMA(0,0,0) with non-zero mean : 7098.224
 ARIMA(1,0,0) with non-zero mean : 7018.447
 ARIMA(0,0,1) with non-zero mean : 4536.125
 ARIMA(0,0,0) with zero mean     : 7090.759
 ARIMA(1,0,1) with non-zero mean : Inf
 ARIMA(0,0,2) with non-zero mean : Inf
 ARIMA(1,0,2) with non-zero mean : Inf
 ARIMA(0,0,1) with zero mean     : 4529.139
 ARIMA(1,0,1) with zero mean     : Inf
 ARIMA(0,0,2) with zero mean     : Inf
 ARIMA(1,0,0) with zero mean     : 7011.515
 ARIMA(1,0,2) with zero mean     : Inf

 Now re-fitting the best model(s) without approximations...

 ARIMA(0,0,1) with zero mean     : 7096.414

 Best model: ARIMA(0,0,1) with zero mean
> Box.test(fit1$residuals^2,lag=12, type="Ljung-Box")

        Box-Ljung test

data:  fit1$residuals^2
X-squared = 200.09, df = 12, p-value < 2.2e-16
```

- Russia

```
> fit1 <- auto.arima(MOEXI.ret, trace=TRUE, test="kpss",  ic="bic")

 Fitting models using approximations to speed things up...

 ARIMA(2,0,2) with non-zero mean : Inf
 ARIMA(0,0,0) with non-zero mean : 4479.893
 ARIMA(1,0,0) with non-zero mean : 4194.135
 ARIMA(0,0,1) with non-zero mean : Inf
 ARIMA(0,0,0) with zero mean     : 4473.822
 ARIMA(2,0,0) with non-zero mean : 4202.272
 ARIMA(1,0,1) with non-zero mean : -24906.68
 ARIMA(2,0,1) with non-zero mean : Inf
 ARIMA(1,0,2) with non-zero mean : Inf
 ARIMA(0,0,2) with non-zero mean : Inf
 ARIMA(1,0,1) with zero mean     : Inf

 Now re-fitting the best model(s) without approximations...

 ARIMA(1,0,1) with non-zero mean : 4493.034

 Best model: ARIMA(1,0,1) with non-zero mean

> Box.test(fit1$residuals^2,lag=12, type="Ljung-Box")

        Box-Ljung test

data:  fit1$residuals^2
X-squared = 47.401, df = 12, p-value = 3.972e-06
```

- India

```
> fit1 <- auto.arima(BSESNI.ret, trace=TRUE, test="kpss",  ic="bic")

 Fitting models using approximations to speed things up...

 ARIMA(2,0,2) with non-zero mean : Inf
 ARIMA(0,0,0) with non-zero mean : 5420.973
 ARIMA(1,0,0) with non-zero mean : 5318.993
 ARIMA(0,0,1) with non-zero mean : Inf
 ARIMA(0,0,0) with zero mean     : 5415.138
 ARIMA(2,0,0) with non-zero mean : 5318.674
 ARIMA(3,0,0) with non-zero mean : 5525.787
 ARIMA(2,0,1) with non-zero mean : Inf
 ARIMA(1,0,1) with non-zero mean : Inf
 ARIMA(3,0,1) with non-zero mean : Inf
 ARIMA(2,0,0) with zero mean     : 5313.332
 ARIMA(1,0,0) with zero mean     : 5312.267
 ARIMA(1,0,1) with zero mean     : Inf
 ARIMA(0,0,1) with zero mean     : 6527.545
 ARIMA(2,0,1) with zero mean     : Inf

 Now re-fitting the best model(s) without approximations...

 ARIMA(1,0,0) with zero mean     : 5421.754

 Best model: ARIMA(1,0,0) with zero mean


> Box.test(fit1$residuals^2,lag=12, type="Ljung-Box")

        Box-Ljung test

data:  fit1$residuals^2
X-squared = 269.6, df = 12, p-value < 2.2e-16
```

- China

```
> fit1 <- auto.arima(SSE.ret, trace=TRUE, test="kpss",  ic="bic")

 Fitting models using approximations to speed things up...

 ARIMA(2,0,2) with non-zero mean : Inf
 ARIMA(0,0,0) with non-zero mean : 6779.22
 ARIMA(1,0,0) with non-zero mean : 6551.526
 ARIMA(0,0,1) with non-zero mean : Inf
 ARIMA(0,0,0) with zero mean     : 6771.696
 ARIMA(2,0,0) with non-zero mean : 6551.044
 ARIMA(3,0,0) with non-zero mean : 6728.932
 ARIMA(2,0,1) with non-zero mean : Inf
 ARIMA(1,0,1) with non-zero mean : Inf
 ARIMA(3,0,1) with non-zero mean : Inf
 ARIMA(2,0,0) with zero mean     : 6545.474
 ARIMA(1,0,0) with zero mean     : 6544.147
 ARIMA(1,0,1) with zero mean     : Inf
 ARIMA(0,0,1) with zero mean     : 2876.236
 ARIMA(0,0,2) with zero mean     : Inf
 ARIMA(1,0,2) with zero mean     : Inf

 Now re-fitting the best model(s) without approximations...

 ARIMA(0,0,1) with zero mean     : 6779.133

 Best model: ARIMA(0,0,1) with zero mean

> Box.test(fit1$residuals^2,lag=12, type="Ljung-Box")

        Box-Ljung test

data:  fit1$residuals^2
X-squared = 1104.7, df = 12, p-value < 2.2e-16
```

- South Africa

```
> Box.test(fit1$residuals^2,lag=12, type="Ljung-Box")    ic="bic")

        Box-Ljung test                                   ...

data:  fit1$residuals^2
X-squared = 299.21, df = 12, p-value < 2.2e-16

 ARIMA(1,0,0) with non-zero mean : 5647.508
 ARIMA(0,0,1) with non-zero mean : 5646.625
 ARIMA(0,0,0) with zero mean     : 5632.884
 ARIMA(1,0,1) with non-zero mean : 5642.761

 Now re-fitting the best model(s) without approximations...

 ARIMA(0,0,0) with zero mean     : 5632.884

 Best model: ARIMA(0,0,0) with zero mean
```

Annex 8: Estimation of VaR by the GARCH model

- Brazil at 1%

```
> report(res_garch01_roll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1854
Data:

==========================================
alpha:                          1%
Expected Exceed:        18.5
Actual VaR Exceed:      32
Actual %:                       1.7%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        8.11
LR.uc Critical:         6.635
LR.uc p-value:          0.004
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        10.497
LR.cc Critical:         9.21
LR.cc p-value:          0.005
Reject Null:            YES
```

- Brazil at 5%

```
> report(res_garch01_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1854
Data:

==========================================
alpha:                          5%
Expected Exceed:        92.7
Actual VaR Exceed:      115
Actual %:                       6.2%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        5.263
LR.uc Critical:         3.841
LR.uc p-value:          0.022
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        5.779
LR.cc Critical:         5.991
LR.cc p-value:          0.056
Reject Null:            NO
```

- Russia at 1%

```
> report(res_garch11_roll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1310
Data:

==========================================
alpha:                          1%
Expected Exceed:        13.1
Actual VaR Exceed:      20
Actual %:                       1.5%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        3.162
LR.uc Critical:         6.635
LR.uc p-value:          0.075
Reject Null:            NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        4.194
LR.cc Critical:         9.21
LR.cc p-value:          0.123
Reject Null:            NO
```

- Russia at 5%

```
> report(res_garch11_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1310
Data:

==========================================
alpha:                          5%
Expected Exceed:        65.5
Actual VaR Exceed:      84
Actual %:                       6.4%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        5.069
LR.uc Critical:         3.841
LR.uc p-value:          0.024
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        6.348
LR.cc Critical:         5.991
LR.cc p-value:          0.042
Reject Null:            YES
```

- India at 1%

```
> report(res_garch10_roll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1840
Data:

==========================================
alpha:                          1%
Expected Exceed:        18.4
Actual VaR Exceed:      72
Actual %:                       3.9%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        90.854
LR.uc Critical:         6.635
LR.uc p-value:          0
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        92.365
LR.cc Critical:         9.21
LR.cc p-value:          0
Reject Null:            YES
>
```

- India at 5%

```
> report(res_garch10_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1840
Data:

==========================================
alpha:                          5%
Expected Exceed:        92
Actual VaR Exceed:      149
Actual %:                       8.1%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        31.562
LR.uc Critical:         3.841
LR.uc p-value:          0
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        31.563
LR.cc Critical:         5.991
LR.cc p-value:          0
Reject Null:            YES
>
```

- China at 1%

```
> report(res_garch01_roll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1824
Data:

==========================================
alpha:                          1%
Expected Exceed:        18.2
Actual VaR Exceed:      49
Actual %:                       2.7%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        35.851
LR.uc Critical:         6.635
LR.uc p-value:          0
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        36.176
LR.cc Critical:         9.21
LR.cc p-value:          0
Reject Null:            YES
>
```

- China at 5%

```
> report(res_garch11_roll, type = "VaR", VaR.alpha = 0.01, conf.level = 0.99)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1965
Data:

==========================================
alpha:                          1%
Expected Exceed:        19.7
Actual VaR Exceed:      50
Actual %:                       2.5%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        33.171
LR.uc Critical:         6.635
LR.uc p-value:          0
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        33.237
LR.cc Critical:         9.21
LR.cc p-value:          0
Reject Null:            YES
```

- South Africa at 1%

```
> report(res_garch01_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1824
Data:

==========================================
alpha:                          5%
Expected Exceed:        91.2
Actual VaR Exceed:      101
Actual %:                       5.5%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        1.073
LR.uc Critical:         3.841
LR.uc p-value:          0.3
Reject Null:            NO

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        4.301
LR.cc Critical:         5.991
LR.cc p-value:          0.116
Reject Null:            NO
```

- South Africa at 5%

```
> report(res_garch11_roll, type = "VaR", VaR.alpha = 0.05, conf.level = 0.95)
VaR Backtest Report
===========================================
Model:                          sGARCH-norm
Backtest Length:        1965
Data:

==========================================
alpha:                          5%
Expected Exceed:        98.2
Actual VaR Exceed:      140
Actual %:                       7.1%

Unconditional Coverage (Kupiec)
Null-Hypothesis:        Correct Exceedances
LR.uc Statistic:        16.596
LR.uc Critical:         3.841
LR.uc p-value:          0
Reject Null:            YES

Conditional Coverage (Christoffersen)
Null-Hypothesis:        Correct Exceedances and
                                Independence of Failures
LR.cc Statistic:        21.468
LR.cc Critical:         5.991
LR.cc p-value:          0
Reject Null:            YES
```

Annex 9: Chi-2 distribution

	p value												
f	**0.995**	**0.99**	**0.975**	**0.95**	**0.9**	**0.75**	**0.5**	**0.25**	**0.1**	**0.05**	**0.025**	**0.01**	**0.005**
1	0.00	0.00	0.00	0.00	0.02	0.10	0.45	1.32	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.21	0.58	1.39	2.77	4.61	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	0.58	1.21	2.37	4.11	6.25	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	1.06	1.92	3.36	5.39	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	2.67	4.35	6.63	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.20	3.45	5.35	7.84	10.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.22	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.39	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.55	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.70	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.85	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	15.98	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	10.17	13.34	17.12	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	11.04	14.34	18.25	22.31	25.00	27.49	30.58	32.80

Source: Passel, 2016