G. S. Wijesiri, “Thetanulls of cyclic curves of genus 4”, Annual research symposium, FGS, University of Kelaniya, Kelaniya, pp.105, November 2013.

Abstract:

Let $\aleph$ be an irreducible smooth projective curve of genus $g > 1$ defined over the complex field $\mathbb{C}$. When the curve is hyperelliptic, the curve has extra automorphism σ and has an equation $y^2 = f(x)$, where x is an affine coordinate on $\mathbb{P}^1$ and σ has order 2. By Hurwitz formula, there are $2g + 2$ branch points $p_1, \cdots, p_{2g+2}$ in $\mathbb{P}^1$ where $\mathbb{P}^1$ denotes the projective line. The problem of expressing branch points in terms of transcendentals (period matrix, thetanulls, etc.,) is an old problem that goes back to Riemann, Jacobi, Picard and Rosenhein. In our previous work [1] we give treatment for the problem of hyperelliptic curves of genus 2 and 3. For genus 2 curves there are 16 thetanulls. In [1] we express the branch points of hyperelliptic curves in terms of the even theta constants and also some relations among even theta constants. Inverting such period map has an application in fast genus 2 curves arithmetic in cryptography. According to Gaudry [2], the cost of scalar multiplication of elliptic curves is twice large as for genus 2 curves and genus 2 cryptosystem is faster than an elliptic curve cryptosystem. Author of [2] uses formulae for the arithmetic in the Kummer surface that comes from the theory of Theta functions. In [1], authors provide formulae that can be used in cryptosystem of genus 2 and genus 3 algebraic curves.

In this paper we determine the hyperelliptic curves of genus 4 in terms of thetanulls and further study relations among the classical thetanulls of such curves $\aleph$. In our work we use formulas for small genus curves introduced by Rosenhein, Thomae’s formulas for hyperelliptic curves, some recent results of Hurwitz space theory, and symbolic manipulation.