T. Shaska, C. Shore, G. S. Wijesiri, “Codes over rings of size p^2 and lattices over imaginary quadratic fields”, Finite Fields Appl. 16 (2010) no. 2, 75-87.

Abstract

Let $\ell > 0$ be a square-free integer congruent to 3 mod 4 and O_K the ring of integers of the imaginary quadratic field $K = \mathbb{Q}(\sqrt{-\ell})$. Codes C over rings O_K/pO_K determine lattices $\Lambda_\ell(C)$ over K. If $p \nmid \ell$ then the ring $R := O_K/pO_K$ is isomorphic to F_{p^2} or $F_p \times F_p$. Given a code C over R, theta functions on the corresponding lattices are defined. These theta series $\theta_{\Lambda_\ell(C)}(q)$ can be written in terms of the complete weight enumerators of C. We show that for any two $\ell < \ell'$ the first $\frac{\ell+1}{4}$ terms of their corresponding theta functions are the same. Moreover, we conjecture that for $\ell > \frac{p(n+1)(n+2)}{2}$ there is a unique symmetric weight enumerator corresponding to a given theta function. We verify the conjecture for primes $p < 7$, $\ell \leqslant 59$, and small n.

Keywords

- Codes;
- Lattices;
- Theta functions