3.15 A model to explain interference patterns using probability density distribution

P.G.T. Harshani, L.N.K.de Silva

ABSTRACT

In classical physics, Young's double-slit experiment is explained in terms of waves passing through the slits. However, in Quantum Mechanics this has to be explained in terms of particles as particles passing through the double-slit one after the other produce the interference patterns. However, so far no model has been developed to explain the interference pattern in terms of particles.

One of the reasons for not developing such a model is the reluctance of the Physicists to believe that a particles passes through both slits at the same time.

In this model, we assume that a photon or an electron passes through both slits. Assuming the symmetrical setup we write down wave function

$$\psi_1(r_1,\theta_1) = \sum_{l=0}^{\infty} \frac{A_l}{r_1^{l+1}} P_l(cos\theta_1) \qquad (1)$$

giving the probability distribution on the screen due to particles passing through the slit 1.

Similarly, writing down

$$\psi_2(r_2,\theta_2) = \sum_{l=0}^{\infty} \frac{B_l}{r_2^{l+1}} P_l(cos\theta_2) \qquad (2)$$

we obtain the probability density distribution for the same particle passing through the slit 2.

Using the principle of superposition we calculate the probability density distribution on the screen for a particle passing through both slits.

Assuming that the double slit is at O, the distance of the screen from O is D, and the angle measured anti-clock wise from the middle of the double-slit to the screen is θ, P the probability density distribution at a point X on the screen is given by

$$
\begin{aligned}
P
&= |A|^2 \sum_{l=0}^{\infty} \left\{ \left\{ \frac{\left\{P_l\left\{\cos\left[\tan^{-1}\left(-{}^{d}/_{D} + \tan\theta\right)\right]\right\}\right\}^2}{(D^2\sec^2\theta + d^2 - 2dD\tan\theta)^{l+1}} \right\} \right. \\
&+ \left\{ \frac{\left\{P_l\left\{\cos\left[\tan^{-1}\left({}^{d}/_{D} + \tan\theta\right)\right]\right\}\right\}^2}{(D^2\sec^2\theta + d^2 + 2dD\tan\theta)^{l+1}} \right\} \\
&+ \left. \frac{P_l\left\{\cos\left\{\tan^{-1}\left(-{}^{d}/_{D} + \tan\theta\right)\right\}\right\} P_l\left\{\cos\left\{\tan^{-1}\left({}^{d}/_{D} + \tan\theta\right)\right\}\right\}}{\left[(D^2\sec^2\theta + d^2 - 2dD\tan\theta)(D^2\sec^2\theta + d^2 - 2dD\tan\theta)\right]^{\left(l + {}^{1}/_{2}\right)}} \right\} \qquad (3)
\end{aligned}
$$

in terms of Legendre Polynomials, where d is the width of the double slit.

We have obtained graphs to demonstrate the variation of general formula (Eq.(3)) of probability density distribution with respect to the angle θ, that resemble the interference patterns.

References:

1.Baggott J.; Meaning of quantum mechanics, A guide for students of Chemistry and Physics, oxford university press

2.Ghatak A. & Lokanathan S.; *Quantum mechanics, Theory and Applications*,Kluwer Academic Publishers

3.Green.N.J.B;Quantum Mechanics 1:Foundations,Oxford University Press,1997