3.18 Brans – Dicke theory and Mach's Principle

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ABSTRACT

In this study, we have considered the application of Brans – Dicke theory in respect of Mach's principle Brans and Dicke have derived the following equation for scalar potential ϕ.

$$R_{ij} - \frac{1}{2} g_{ij} R = (8\pi\phi^{-1}/c^4) T_{ij} + (\omega/\phi^2)\left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k}\right) + \phi^{-1}(\phi_{,i;j} - g_{ij}\Box\phi) \quad (1)$$

that is needed to describe the universe in addition to the tensor field in the usual Einsteinian theory.

The usual Friedman equations from the Einsteinian theory are given below

$$8\pi G\rho = \frac{3}{R^2}\left(kc^2 + \dot{R}^2\right) - \Lambda \qquad (2)$$

$$\frac{8\pi G}{c^2} p = -\frac{2\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{kc^2}{R^2} + \Lambda \qquad (3)$$

Brans and Dicke have quoted Sciama's condition

$$GM/Rc^2 \sim 1$$

as a condition that should be satisfied by G (gravitational constant), M (Mass of the universe) in order to fulfill Mach's principle.

By considering $= \left({}^{9}/_{2}\right)^{1/3} ct^{2/3}$, one of the solutions of Friedman equations as the background universe, we find that even though Sciama's condition is satisfied the gravitational constant terns out to be negative. This implies that though Sciama's condition is a necessary condition for the fulfillment of Mach's principle, it is not a sufficient condition.

References:

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