Residual properties of groups

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ABSTRACT

A group G is called a residually P group if, for eachb non-identity element g in G, there exists an epimorphism f from G to a group in P such that the image of a under f is not the identity in ZP. There are several other equivalent definitions also.

One important and interesting property worthy of investigation is that of residual finiteness. Accordingly, a group G is residually finite, if for each $1 \neq g \in G$, there is an epimorphism f from G to a finite group A (say) such that $f(g) \neq 1$ in A. From the very definition, it is clear that all finite groups are residually finite. Further, the cycle group of integers under addition is residually finite. It can be proved that subgroups as well as direct products of residually finite groups are residually finite. Also, any free group, any polycyclic group and any subgroup of a general linear group GL (n,F) are residually finite. Nevertheless homomorphic image of a residually finite group need not be residually finite. Further, a

- Finitely presented residually finite group has a solvable word problem
- Finitely generated residually finite group is Hopfiam
- Group of automorphisms of a finitely generated residually finite group is residually finite.
- Free product of two residually finite groups is residually finite.
- Free product of tow finite groups amalgamating only one subgroup is residually finite.

A simple example of a group that is not residually finite is the additive group of rationales.

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