3.10 Tests against tree order restriction in Poisson intensities

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ABSTRACT

We consider a situation in which one wishes to compare several Poisson intensities with a control or standard when it is believed that the intensities are higher than the control.

For instance, if λ_1 is the average accident rate per k.m. of a truck driver who has undergone an extensive training in driving and if λ_j, for j=2,3,...,k, are the average accident rates of the jth truck drivers without any prior training; and if the intensities are believed to produce at least as large an intensity as the control, then one would expect that $\lambda_1 \leq \lambda_j$ for j=2,3,...,k. If x_i is the number of accidents incurred by the ith of the k truck drivers and t_i be the number of k.m. he drove and λ_i be the average accident rate per k.m., we can mathematically formulate this situation as follows:

Suppose $X_1,...,X_k$ are independent Poisson variables with means $\mu_i = \lambda_i t_i$ and let H_o: $\lambda_1 = \lambda_2 = ... = \lambda_k$ and H_1: $\lambda_1 \leq \lambda_j$, for j=2,3,...,k, where λ_1 is the control intensity and λ_j, for j =2,3,...,k, are the other intensities. The ordering specified by H_1 is called a tree ordering. We are interested in testing H_o versus H_1-H_o. The likelihood ratio test for H_o versus H_1-H_o is computed and we derive the asymptotic distribution of it.

Robertson and Wegman (1978) considered order restricted tests for members of the exponential family. Their results can be applied in the testing situation considered here only if the t_i are all equal. Some results are also obtained in the literature for other order restrictions (Magel & Wright (1984) and Barmi *et al*.(1996)). In this study we obtain explicit formulae for the null distribution of the test statistic under tree ordering.

References
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