

4.9 Cosmological Models with Both Acceleration and Deceleration

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ABSTRACT

Since Perlmutter and others (1997) & (1998)^{1,2} observed that the universe expand with an acceleration, many models involving dark energy have been proposed to explain this phenomenon. In this paper we present a family of cosmological models with both acceleration and deceleration .

We write Einstein's Field Equations in general relativity in the form,

$$G^{\mu\nu} = \kappa T^{\mu\nu} + \wedge g^{\mu\nu} , \text{ where } \kappa = -\frac{8\pi G}{c^2} .$$

The \wedge term introduced by Einstein himself gives rise to a field that repels particles and objects rather than to one that attracts them. Hemantha and de Silva (2003)&(2004)^{3,4} modified the field equations so that what is conserved is not the energy momentum of matter and radiation but the energy momentum of matter and radiation and the energy of the \wedge field, which they considered as the "dark energy". They obtained the equations,

$$\begin{aligned} \kappa p &= \wedge c^2 + \frac{kc^2}{R^2} + \frac{\dot{R}^2}{R^2} + \frac{2\ddot{R}}{R} \\ \kappa \rho &= -\wedge - \frac{3k}{R^2} - \frac{3\dot{R}^2}{R^2 c^2} , \end{aligned}$$

where $\dot{}$ denotes differentiation with respect to cosmic time t . The above equations lead to

$$3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{R}}{R} + \dot{\rho} + \frac{\dot{\wedge}}{\kappa} = 0$$

As the density $\rho(t)$ has to be a positive quantity we can show that $k = 1$, is the only possible value of k that satisfies the above equations.

We assume that a family of solutions of above equations for R , can be written in the form ,

$$R = a + b_1 \cos \omega t + b_3 \cos 3\omega t .$$

Using the boundary conditions,

$$* R = 0 \text{ at } t = 0 .$$

$$* \text{ and } \dot{R} = 0, \ddot{R} = 0, \text{ at } t = \frac{\pi}{2}, \text{ (point of inflection)}$$

we have
$$R = -b_3(1 - \cos^3 \omega t) .$$

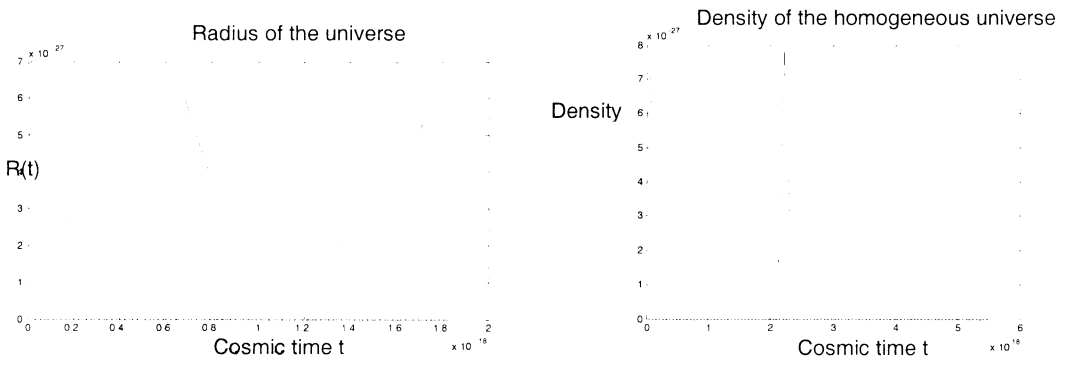
Recent observations⁵ have led to the approximate value $\frac{7}{3}$ for the ratio of dark energy

$$\left(\Lambda' = -\Lambda c^2 / 8\pi G \right) \text{ to matter density } (\rho) \left(\frac{\Lambda'}{\rho} \Big|_{t=t_0} \right) , \text{ and to the value } \underline{1.6} \text{ for the}$$

$$\text{redshift } \left(\frac{R|_{\omega t = \omega t_0}}{R|_{\omega t = \frac{\pi}{2}}} \right) , \text{ at the onset of acceleration. Taking this redshift to be a constant}$$

a family of solutions can be found for different ratios of dark energy to matter. Similarly keeping the ratio of dark energy to matter as $\frac{7}{3}$ we find that a family of solutions can be obtained for different values for the above redshift. Though there is no solution when the redshift is 1.6, there is a solution when its value is 1.3, which is good enough considering the uncertainties associated with measurements.

The age of the universe is estimated⁶ to be 13.7 billion years. Then taking the present value of the cosmic time t as 13.7 billion years, we find $b_3 = -8.33 \times 10^{26} \text{ cm}$, $\omega = 5.16 \times 10^{-18} \text{ rad } t^{-1}$, when the above redshift is 1.3. The graphs for these values are given below. It is seen that $R(t)$ has both acceleration and deceleration.



References :

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5. Perlmutter S. Supernovae Dark Energy , and the Accelerating Universe, Physics Today , April 2003 , 57.
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