4.3 The Schwarzschild Space-Time in the Background of the Flat Robertson-Walker Space-Time

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ABSTRACT

The Schwarzschild space-time is well known in describing the gravitational field of an object in an otherwise empty universe. The Schwarzschild space-time was derived by Karl Schwarzschild (1916) considering the merger of the Schwarzschild space-time with the Lorentz metric as the boundary [1]. However, the Lorerntz metric cannot be used in investigations of non empty large scale space-times, the whole universe being one such case. Thus, the cosmologists use the Robertson-Walker space-times, in describing the universe [2, 3]. As a result it becomes necessary to investigate the gravitational field of an object in the background of the Robertson-Walker space-time,

$$ds^2 = c^2\ dt^2 - \frac{R^2(t)}{\left(1 + \frac{kr^2}{4}\right)^2}\left\{dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right\}.$$

We have studied the merger of the isotropic Schwarzschild space-time with the flat Robertson-Walker space-time. In this scenario, the flat Robertson-Walker space-time was considered for simplicity. The expressions for the radial coordinates r_μ and $\bar{r}_\mu$ at the merger of the flat Robertson-Walker space-time and the isotropic Schwarzschild space-time were derived in terms of the scale factor $R(t)$ and a constant R^* and found to be given by

$$r_\mu = \left(\frac{m}{2}\right)\left[\frac{1}{\sqrt{R^*}\left(\sqrt{R(t)} - \sqrt{R^*}\right)}\right] \text{ and } \bar{r}_\mu = \left(\frac{m}{2}\right)\left[\frac{\sqrt{R^*}}{\left(\sqrt{R(t)} - \sqrt{R^*}\right)}\right].$$

An analytic expression for the time coordinate ($\bar{t}$) of the Schwarzschild space-time was obtained in the case of the de-Sitter universe,

$\bar{t} = 2T_0 \, ln\left[\dfrac{\sqrt{R^*}}{2\sqrt{R^*} - \sqrt{R(t)}}\right]$, where T_0 is the reciprocal of the Hubble constant [2].

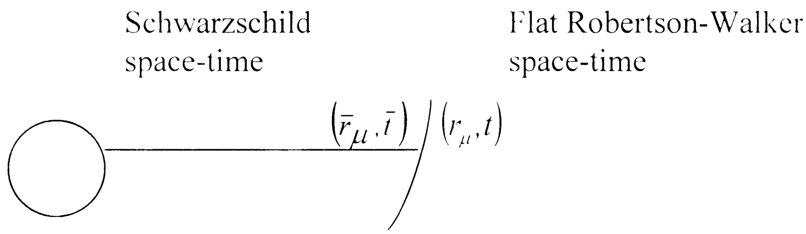

Figure: The radial coordinates and the time coordinates of the Schwarzschild space-time and the flat Robertson Walker space-time at the merger

The derived expressions for the radial coordinates r_μ and $\bar{r}_\mu$ imply that an object in the universe begins to communicate with the "outside world" after a particular time, before which r_μ and $\bar{r}_\mu$ are negative. At this particular time, $R(t)$ approaches the constant R^* and r_μ , $\bar{r}_\mu$ tend to infinity. It could be said that the object comes into existence as far as the rest of the universe is concerned at this particular instant. The values of r_μ and $\bar{r}_\mu$ decrease with increase of time. When the time coordinate of the Schwarzschild space-time tends to infinity, $\bar{r}_\mu$ achieves the value $\left(\dfrac{m}{2}\right)$, the value of the Schwarzschild radius in isotropic coordinates.

References:

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2. McVittie, G.C.; *General relativity and Cosmology*; Chapman and Hall Ltd., 11 New Fetter Lane London; **1964.**
3. Shiromizu, T.; Gen, U.; *Space-times which are asymptotic to certain Friedman-Robertson-Walker space-times at time like infinity*; **2006,** gr-qc/ 9905092v1.