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Paper: Transformability

On the velocity of waves in Quantum Mechanics

It is generally believed that in Classical Physics it is the group velocity of a wave that carries information from one point to another point in space. The group velocity under normal circumstances for classical waves turns out to be less than that of light and the phase velocity though could be greater than the velocity of light, is not believed to carry information. However, in the case of Quantum Mechanical de Broglie waves corresponding to particles we could obtain an expression for the phase velocity in terms of the momentum (hence the velocity of the particle or the group velocity of the de Broglie wave), which is of more significance as far as Quantum Mechanical particles (systems) are concerned.

Consider a particle of mass m moving with velocity v in a frame of reference F, and suppose that it exhibits Quantum Mechanical properties. If E is the energy of the particle and $p= mv$ is its momentum in F then the de Broglie wave length and the corresponding frequency are given by $\lambda = h / p$ and $\omega = u / \lambda$ respectively where u is the phase velocity of the de Broglie wave. Since $E = h\,\omega$ we have $E=up$. Substituting these in the relativistic equation $E^2/c^2 = p^2 + m_0^2c^2$ we have $p^2 (u^2/c^2 -1) = m_0^2c^2$ and $m_0^2 v^2(u^2/c^2 -1)/(1-v^2/c^2)= m_0^2c^2$. These equations imply that $u>c$ and $uv=c$. It can be seen that the group velocity turns out to be v, the velocity of the particle. Thus the phase velocity u of the de Broglie wave is $c/v= mc/p$, in terms of p the momentum of the particle.

Now let the frame F be moving with velocity w in a frame of reference F^1 in the same direction as that of v. Then the velocity v^1 of the particle in F^1 is given by the addition formula $v^1 = (v+w) / (1+vw/c^2)$. If u^1 is the phase velocity of the de Broglie wave as observed in F^1 then $u^1v^1=c^2$. This gives $u^1= (u+w)/(1+uw/c^2)$ for the phase velocity of the de Broglie wave in frame F^1 agreeing with the usual special relativistic law of addition of velocities. For photons both the phase velocity and group velocity turn out to be c for any frequency, and for particles when $v<c$, the phase velocity u turns out to be greater than c. Though in the case of Newtonian particles information is not carried with phase velocity it may not be the case in Quantum Mechanical particles.

Contd.

The Aspect (1981, 1982a, 1982b) experiments carried out to resolve the EPR paradox showed that some kind of interaction between "entangled" particles took place at velocities exceeding that of light. While the system of particles could be considered as one system and assumed that interactions take place instantaneously as Bohr maintained to the dissatisfaction of Einstein who called such interactions spooky actions[i], it may also be argued that in the case of "entangled" particles one particle communicates with the other at a velocity greater than that of light given by the phase velocity as measured by the relative velocity of the two particles, and that the other particle is informed of the measurement after a finite time depending on the distance between the particles and the phase velocity. It should be possible to design an experiment to measure the velocity of communication between such particles.

We conclude that in the case of Quantum Mechanical particles, waves carry information with velocities equal to that of the phase velocities calculated on the basis of the de Broglie waves. Instantaneous communication with infinite velocities is not possible as suggested by Bohr, as it demands zero relative velocity of the particles. In Quantum Mechanics, a particle cannot be at rest at a given place as it violates the uncertainty principle and hence communication at infinite phase velocity is not possible. Communication with finite velocities calculated on the basis of the phase velocity, though greater than that of light, in the case of Quantum Mechanical entangled particles seems to be a way out of spooky action.

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