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Poster

Convergence properties in numerical algorithms of option price calculations

Financial derivatives can be used to minimize losses caused by price fluctuations of the underlying assets where derivative means financial instrument whose value depends on the price of some other financial assets or some underlying factors. Options are widely used on markets and exchanges. Options can be used, for instance, to hedge assets and portfolios in order to control the risk due to movements in the share price.

The famous Black-Scholes model is a convenient way to calculate the price of an option. And also the Black–Scholes model [1] is a mathematical description of financial markets and derivative investment instruments. In an idealized financial market the price of the European option can be obtained as the solution of the celebrated Black-Scholes equation. This equation also provides a hedging portfolio that replicates the contingent claim. However, the Black-Scholes equation has been derived under quite restrictive assumptions (for instance, frictionless, liquid, complete markets). In recent years, some of these restrictive assumptions have been relaxed in order to model, for instance, the presence of transaction costs, imperfect replication and investor's preferences, introduction of a given stock-trading strategy of a large trader, and risk from unprotected portfolio. These models lead to a generalized Black-Scholes equation for the price of an option in which the volatility need not be necessarily constant and it may depend on the asset price as well as the option price. Again, if transaction costs are taken into account then the classical Black-Scholes theory is no longer applicable. In order to maintain the delta hedge one has to make frequent portfolio adjustments yielding thus a substantial increase in transaction costs.

However, these modifications convert the Black-Scholes equation into non-linear form and it is very difficult to solve the equation analytically. In the absence of analytical methods, various numerical discretization methods can be used in order to approximate the solution of the Black-Scholes equation. In our conditions we investigate the numerical solution of the Black-Scholes equation for the European call option,

$$u_\tau = S_i^n\left(u_{xx} + u_x\right) + \left(1 + D\right)u_x + u_{xx}$$

where $S_i^n = \left(\sqrt{\dfrac{2}{\pi}}\right)\dfrac{K}{\sigma\sqrt{\delta t}}\, sign\left(u_{xx} + u_x\right),\ D = \dfrac{2r}{\sigma^2}$, K is strike price, σ is volatility, r is interest rate.

01.Bertam Dauring, Fachbereich Mathematic and Statistik, University Konstanz, Germany, MichelFournie, University Paul Sabatier,France, *A nonlinear Black-Scholes equation which models to high order compact Finite difference schemes for a nonlinear Black-Scholes equation*,2004