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Poster

Possible quark confinement by a non-relativistic model

Confinement of quarks by an infinitely deep potential well is well [1] known. We are interested in confinement of quarks by the singular potential $-\frac{\alpha_1}{r^2}$ when the effective potential , $\left(\frac{l(l+1)}{r^2}-\frac{\gamma}{r^2}\right)$ is negative, where $\gamma=\frac{2\mu\alpha_1}{\hbar^2}$. However, we have found that the corresponding series solution is not a bound wave function. Now, we assume that quarks are localized to a small region and obtain the bound states in the following way.

Consider the Schrödinger equation in the form

$$\frac{d^2u}{dr^2}+\left(-k^2-\frac{l(l+1)}{r^2}+\frac{\gamma}{r^2}\right)u=0 \tag{1.1}$$

and let us choose γ such $\gamma=l(l+1)$. Then (1.1) reduces to $\frac{d^2u}{dr^2}-k^2u=0$

and the wave function $u(r)=e^{-kr}$ and the total radial wave function is given by

$$R(r)=\frac{u(r)}{r}=\frac{e^{-kr}}{r} \tag{1.2}$$

which is normalizable and the normalization constant $(2k)^{\frac{1}{2}}$. We conclude that non relativistic quarks having nonzero angular momentum can be bound by the inverse square potential and the quark wave function can be made highly localized acquiring sufficient energy $\frac{\hbar^2k^2}{2\mu}$. We use the experimental value of the size(diameter) of the nucleon of $1.6\,fm$ to determine the value of k. We can attribute this value to the mean square radius given by

$$<r^2>=\int_0^\infty 2kr^2e^{-2kr}dr=<\psi\,|\,r^2\,|\,\psi>=0.64 \tag{1.3}$$

The equation (1.3) gives

$$k^2=\frac{1}{2\times(0.64)} \tag{1.4}$$

We have assumed that the quark mass is μ and therefore the quark binding energy E is given by $\frac{\hbar^2k^2}{2\mu}$,

and if we use μ to be one third of the nucleon mass , then

$$E = -\frac{197.\times 197\times 3k^2}{2\times 938} \approx -62k^2 = -48.437MeV$$

$$< r >= \frac{1}{2k} = 0.556\ fm$$

Strength of the potential can be found in this case by using $l(l+1)=\gamma$. If $l=1$, $\frac{2\mu}{\hbar^2}\alpha_1 = \gamma = 2$.

Therefore $\alpha_1 = \frac{.\hbar^2}{\mu} \approx \frac{3.200.200}{938} \approx 60MeV$

Another important point to be mentioned here is that attractive potential can be bound to potential centre of a circular orbit by an inverse square potential only if the total energy of the particle is zero in case of classical mechanics. Therefore, our quark bound states might be stable if they are confined to a very small region and they are undisturbed. This conclusion is actually based on classical mechanics but plausible since speeds of quarks should be big.

Reference

[1] Models of the Nucleon From Quarks to Soliton. Rajat K. Bhaduri, Addison- Wesley Publisher , INC(1988)