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### Advanced plane geometry research - II

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Advanced plane geometry has been the most complicated or intricate sector in pure mathematics since the history of mathematics as most of the correlations in plane geometry cannot be discovered as easily as in other sectors of mathematics such as algebra, calculus etc because plane geometry is composed of lines, circles and triangles etc. A general mathematician has to have an in-depth analysis to detect relationships, not only among lines but also among the expressions or equations used in algebra and calculus, besides the analysis of geometrical lines. Therefore, nearly entire sections in pure mathematics have to be used to solve questions or relationships in advanced plane geometry which is what prevents the discovery of new relationships in advanced plane geometry and the lack of interest of mathematicians in plane geometry. Therefore, it is hoped that some of the following relationships will contribute to the existing knowledge in plane geometry.

(1) ABC is any triangle as  $AB=c, BC=a, CA=b$ . AD is the internal bisector of angle A which meets BC at D. E and F are points on AB and AC respectively. AD, BF and CE are concurrent at O as ED and FD are perpendicular at D. The relationship is that when above given conditions are satisfied, DE and DF are the bisectors of angle ADB and angle ADC respectively. This is a very considerable and significant relationship obtained by myself regarding angle bisectors in this research as this relationship can be used very widely throughout many sections in plane geometry.

(2) ABCD is any quadrilateral as E and F points are marked as  $BE=FC$  on its BC diagonal. The relationship is that  $AB^2+AC^2+DE^2+DF^2=BD^2+DC^2+AE^2+AF^2$ . When ABC and BDC are right angled triangles as angle  $BAC = \text{angle } BDC = 90^\circ$  then  $AE^2+AF^2=DE^2+DF^2$  which implies that if any E and F points are marked as  $BE=FC$  when the hypotenuses of any two right angled triangles coincide as ABC and BDC right angled triangles  $AE^2+AF^2=DE^2+DF^2=K$  (K is a constant). It should also be mentioned that the correlation for ABC triangle  $AB^2+AC^2=AE^2+AF^2+2BE \cdot BC$  which is proved to obtain above relationships is valid for any ABC triangle when  $BE=FC$ .

(3) PQRS is an any quadrilateral as UZ and VZ are met on Z on the extended SQ length. U and V are any points of PS and SR while UZ and VZ cut PQ at X and QR at T. The relationship is that if UV is parallel with PR then UV is also parallel with XT. The significant converse of this correlation is that if UV is parallel with XT and PR then UX and VT are met on a point at extended SQ line which predicates that the path of lines such as UX and VT as XT, PR, UV are parallel to each of them is the extended straight line SQ.