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The equality of Schrödinger's Theory and Heisenberg's S-matrix Theory

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The main aim of this work is to show that the energy discrete eigen values given by the Schrödinger's theory and Heisenberg's theory are the same. To obtain this result, we have used Parabolic co-ordinates to solve the Schrödinger's equation for the Hydrogen Atom. By using the Hyper Geometric Confluent functions we have expressed the S-matrix element using Gamma functions;

$$S_l^n(k) = \frac{\Gamma(l+1+in)}{\Gamma(l+1-in)} \text{ where } n = -\frac{\mu e^2}{\hbar^2 k}$$

By the definition of Gamma function,

$$S_l(n) = \frac{\bar{z}}{z} e^{\gamma(-2in)} \prod_{p=1}^{\infty} \left\{ \frac{\left(1 + \frac{\bar{z}}{p}\right)}{\left(1 + \frac{z}{p}\right)} e^{2in/p} \right\}$$

Then it is apparent that the S-matrix element contains infinite number of poles and zeros. Considering the relevant simple pole, we have derived an equation for the energy eigen values of the form

$$E_n = -\frac{\mu e^4}{2\hbar^2 n^2}$$

This shows that it is the same as the equation we have obtained in Schrödinger's theory. Therefore Heisenberg's S-matrix theory and Schrödinger's wave mechanics give exactly the same eigen values in the cases we have examined.